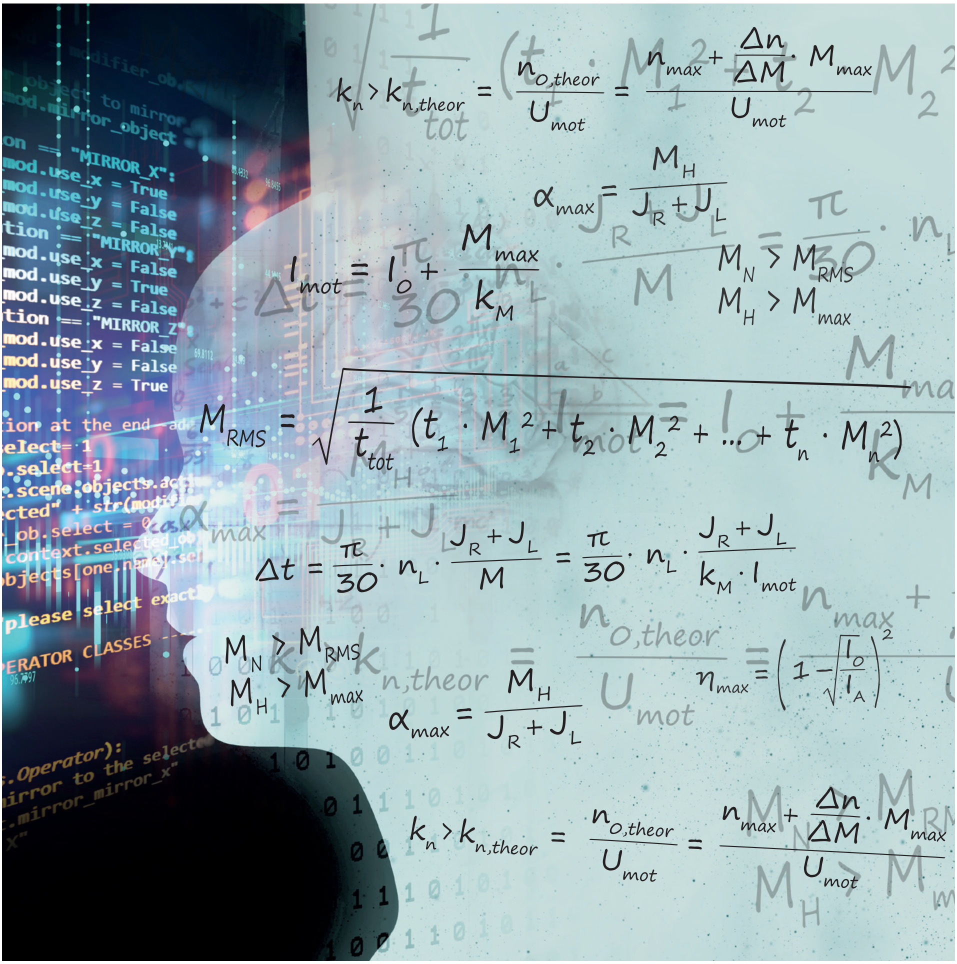
Precision Drive Systems

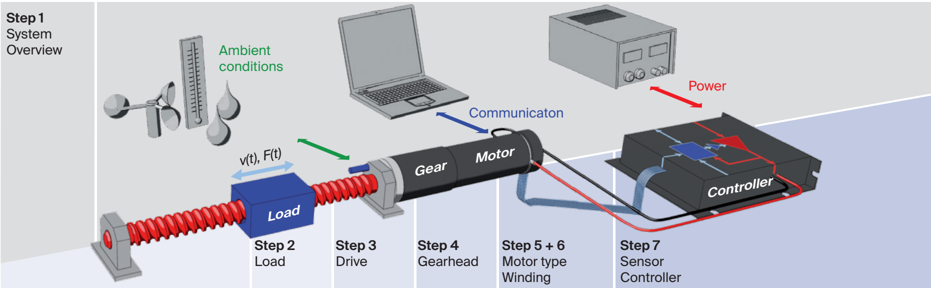
maxon

# Formulae Handbook

maxon academy



# Selection process




When performing a **system analysis**, the first step is to describe the drive as a whole in its environment. The objective is to obtain an **overview** of the system, to determine the theoretical feasibility of a solution and to get a picture of the boundary conditions and restrictions. **See Chap. A.1: Overview, system analysis**

The goal of "The goal of the Motion of the load" is to define the key requirements regarding forces (torques) and velocities (speeds of rotation). How long must they be applied? What is the required control accuracy? **See Chap. A.2: Motion of the load**

The mechanical drive design can be skipped if the load is driven directly and the drive system does not include a **mechanical drive.**

**Mechanical drives** transform mechanical power into mechanical power. For the selection of the drive the load key data are converted to the output of the motor or gearshaft. **See Chap. 3 Mechanical drives.**

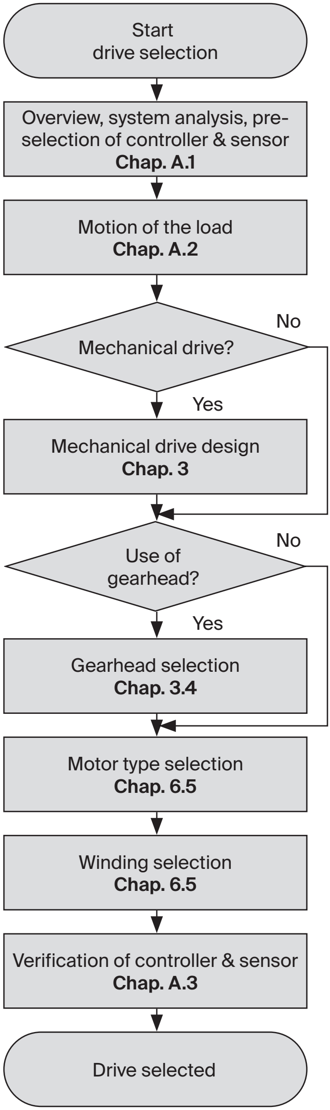
The step for the **gearhead selection** can be skipped if no (maxon) gearhead is used. Gearheads are typically used whenever high torques are required at low speeds.

The purpose of this step is to determine if and which **maxon gearhead** can be used. The key data for the motor selection can then be calculated from the gearhead reduction and efficiency. **See Chap. 3.4: maxon gearhead**

On the basis of the torque and speed requirements, the next step is to select suitable **types of motors**. The useful life, commutation and bearing systems also have to be considered. **See Chap. 3.4: Motor selection**

The **selection of the winding** is made on the basis of a comparison of the applied motor voltage with the speed and a comparison of the available current with the torque requirements. **See Chap. 6.5: Motor selection**

The purpose of the last step is the **verification of the controller and sensor,** as well as a verification that the controller and sensor preselected in the situation analysis (Step 1) are compatible with the selected motor. **See Chap. A.3: Verification of controller and sensor**


Start
drive selection
Overview, system analysis, pre-
selection of controller & sensor
Chap. A.1
Motion of the load
Chap. A.2
Mechanical drive?
No
Yes
Mechanical drive design
Chap. 3
Use of
gearhead?
No
Yes
Gearhead selection
Chap. 3.4
Motor type selection
Chap. 6.5
Winding selection
Chap. 6.5
Verification of controller & sensor
Chap. A.3
Drive selected

## *Foreword*

*This Formulae Handbook lists the most important formulae in relation to all components of the drive system. It makes use of a flow chart that supports quick selection of the correct drive. Numerous illustrations and the clear descriptions of the symbols on the respective page help the reader to understand the formulae.*

*Roughly speaking, it is a collection of the most important formulae from the maxon catalog, as well as from the book "The selection of high-precision microdrives", published by maxon academy.*

*The initiative for writing this Formulae Handbook was the book "The selection of high-precision microdrives" by Dr. Urs Kafader, which contains extensive know-how from the success story of 50-years of maxon DC drives with low power (below approx. 500 W). The collection is intended for engineers, professors, lecturers and students, as a perfect supplement to the above mentioned book.*


## *Thank you*

*Firstly I would like to thank Dr. Urs Kafader, who encouraged me to tackle this book. The professional layout and illustrations were done by Patricia Gabriel and Beni Anderhalden. Urs Kafader, Barbara Schlup, Anja Schütz, Patrik Gnos, Stefan Baumann, Martin Rüegg, Michael Baumgartner, Martin Windlin, Jens Schulze, Albert Bucheli, Martin Odermatt and Walter Schmid have read the manuscript and have given valuable suggestions for improvements. I also received extensive and ready support from many other people at maxon motor ag in response to my questions and requests for assistance.*
*Special thanks go to Susan Bechtiger, Paul Williams, Robin Philips, Anthony Mayr and Mark Casey who helped improve the translation from German into English.*


*Sachseln, Spring 2012*
*Jan Braun*

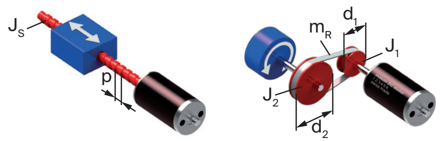

5th Edition 2019

# Content

# A. Drive selection
## A.1 Overview, system analysis

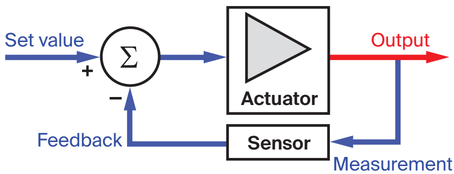
Before the actual selection process begins, a consideration with the drive system in its entirety is needed. The possible range of variations of the key parameters must also be determined. As a rule, all of these aspects are closely interlinked. The descriptions below are intended to help clarify these points and establish a framework for the further selection process.

### Mechanical design



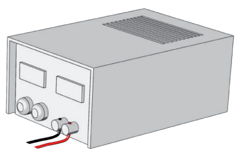

Is the intended motion linear or a rotary? What drive (screw, toothed belt, etc.) or what combination of drives are going to be used to achieve the desired motion? Is it a direct drive?

### Define the control concept



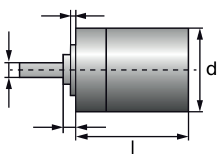

What variables are to be controlled: current, speed, position? With what accuracy? Is an open-loop control system sufficient? How is the controlled variable measured? Where do the commands and set values come from? The answers to these questions will result in a preselection of possible controllers and sensors, i.e. for selection step 7 (see page 2).

### Verify the power components



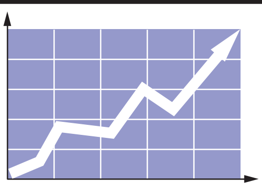
Is sufficient electric power available to drive the load under all operating conditions and to compensate for the expected losses in the drive train? What are the maximum voltage and current that will be available?

### Determine the boundary conditions




Are there restrictions on size? In what environment (temperature, atmosphere etc.) is the drive required to operate? Is compliance with particular specifications or quality standards required? What is the specified useful life?

### Cost considerations



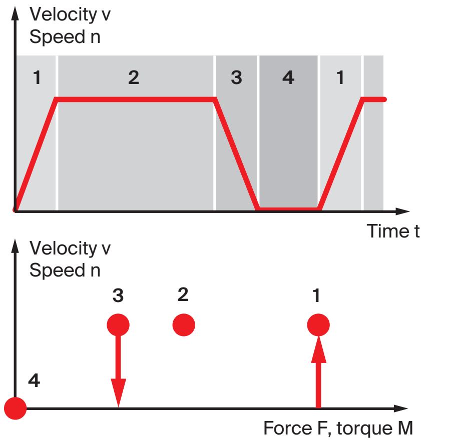
Cost is always a key consideration. How can the drive be designed as economically as possible and still meet the requirements regarding performance and useful life?

For detailed information, refer to the book "The selection of high-precision microdrives", chapter 3.

## A.2 Motion of the load

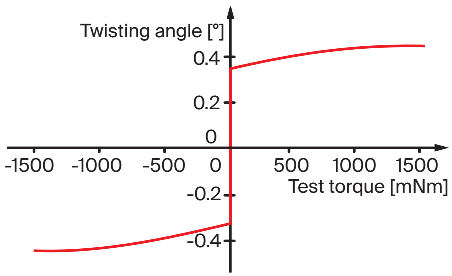
In the step for determining the **load requirements**, the motions to be executed must be defined. It is important to select appropriate motion profiles and to consider which operating times are to be expected.

### Operating points



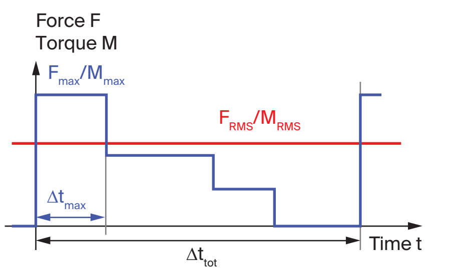

For determining the associated **operating points** (value pairs of torque and speed of rotation or of force and velocity), the respective total forces and torques are important. To this end, all acting forces and torques have to be determined. These in turn depend on the moments of inertia and the acceleration values.For the purpose of making a selection, calculating these values with an accuracy of approx. 10% is sufficient.

### Mechanical play of the drive




Furthermore, the question of the maximum permissible **mechanical play of the drive** has to be determined.

### Key data



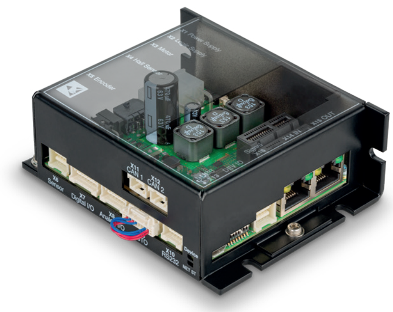

The **key data** which caracterize the load can finally be calculated from the operating points. They are important for selecting the drive.

For detailed information, refer to the book "The selection of high-precision microdrives", chapter 4.

## A.3 Verification of controller and sensor

The **controller and sensor** verification involves checking whether the preselection made during the system analysis (selection step 1, see page 2) are compatible with the motor chosen. Detailed examination of the configuration of the control circuit allows to make definitive decisions regarding the suitable components (controller and sensor).

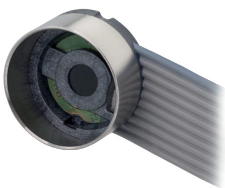
### Motion controller



In higher-level drive systems, the motion controller is the central element. It is where all the threads come together. Thus, the controller must satisfy a wide range of requirements.

The controller must
- be able to control the manipulated variable with sufficient accuracy in a reasonable amount of time
- be able to process the information provided by the sensor
- understand the set values and commands of the higher-level system
- provide the required electric power
- be suited to the motor type (brushed or brushless) and the commutation

### Sensor



The sensor (encoder, DC tacho or resolver) must be appropriate for the control task and comply with the other components. Additionally, the following further selection criteria apply.

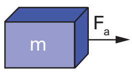
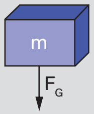
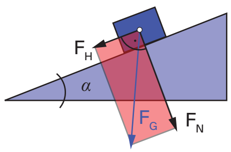
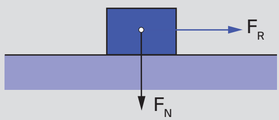
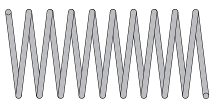
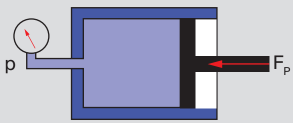
The sensor has to
- be mountable on the motor according to the maxon modular system.
- measure the correct control variable (speed, position, direction of rotation) with sufficient resolution. Rule of thumb: The resolution of the sensor should be at least four times higher than the specified accuracy of the control variable.

For detailed information, refer to the book "The selection of high-precision microdrives", chapter 9.

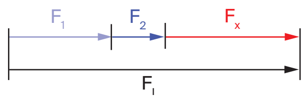
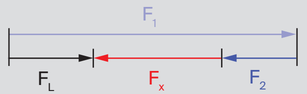
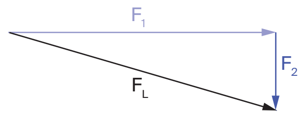
## 1. Mass, force, torque
### 1.1 Forces in general

The force required to accelerate a mass of 1 kg by 1 m/s in 1 s has the unit kg · m/s², with the special unit name Newton (N).

**Typical component forces in a drive system**

| | | |
|---|---|---|
|  | Force for acceleration =<br>mass · acceleration<br>$[F]$ = kg · m/s² = kgm/s² = N | $F_a = m \cdot a = m \cdot \frac{\Delta v}{\Delta t}$ |
|  | Gravitation<br>(gravitational acceleration<br>$g$ = 9.81 m/s² = 9.81 N/kg ≈ 10 N/kg) | $F_G = m \cdot g$ |
|  | Forces on the inclined plane:<br>Downhill-slope force and normal force | $F_H = F_G \cdot \sin\alpha$<br>$F_N = F_G \cdot \cos\alpha$ |
|  | Friction force<br>Sliding friction | $F_R = \mu \cdot F_N$ |
|  | Spring force, compression and extension springs | $F_S = k \cdot \Delta l$ |
|  | Compressive force | $F_p = p \cdot A$ |

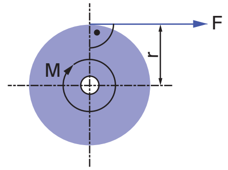
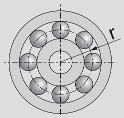
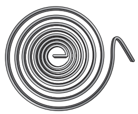
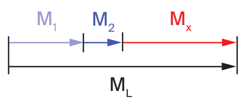
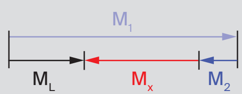
| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $A$ | Cross section | m² | $a$ | Acceleration | m/s² |
| $F$ | Force | N | $g$ | Gravitational acceleration | m/s² |
| $F_a$ | Acceleration force | N | $k$ | Spring constant | N/m |
| $F_G$ | Weight of a body | N | $m$ | Mass | kg |
| $F_H$ | Downhill-slope force | N | $p$ | Pressure (1 Pa = 1 N/m² = $10^{-5}$ bar) | Pa |
| $F_N$ | Normal force<br>(force perpendicular to the plane) | N | $\alpha$ | Angle of the inclined plane | ° |
| | | | $\Delta l$ | Displacement | m |
| $F_p$ | Compressive force | N | $\Delta t$ | Duration | s |
| $F_R$ | Friction force | N | $\Delta v$ | Velocity change | m/s |
| $F_S$ | Spring force | N | $\mu$ | Coefficient of friction (see table Chap. 10.2) | |

| Calculating the total load force consisting of component forces | | |
|---|---|---|
|  | Addition of forces acting in the same direction | $F_L = F_1 + F_2 + ... + F_X$ |
|  | Addition of forces acting in opposite directions | $F_L = F_1 - F_2 - ... - F_X$ |
|  | Addition of perpendicular forces | $F_L = \sqrt{F_1^2 + F_2^2}$ |

| Symbol | Name | SI |
|---|---|---|
| $F_L$ | Load force (output) | N |
| $F_1 / F_2 / F_x$ | Partial forces | N |

## 1.2 Torques in general

The torque is a measure of the rotational effect that a force exerts on a rotating system. It plays the same role for rotation that the force plays for linear motion. The equations always apply for a defined axis of rotation.

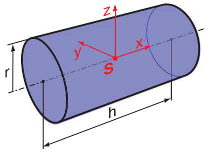
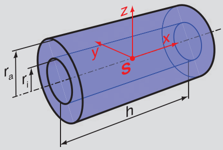
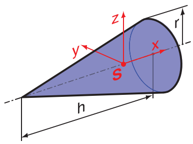
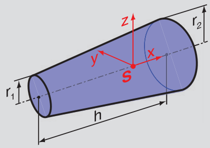
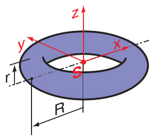
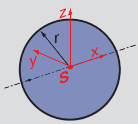
| General | | |
|---|---|---|
|  | Torque = force · lever arm<br>$[M]$ = N · m = Nm | $M = F \cdot r$ |
| **Typical component torques in drive systems** | | |
| Torque for acceleration of moments of inertia<br>Torque = moment of inertia · angular acceleration<br>(For information on calculating moments of inertia, see the next pages) | | $M_\alpha = J \cdot \alpha = J \cdot \frac{\Delta\omega}{\Delta t}$<br>$M_\alpha = J \cdot \frac{\pi}{30} \cdot \frac{\Delta n}{\Delta t}$ |
|  | Friction of ball bearing and sintered sleeve bearing (simplified) | $M_R = \mu \cdot F_{KL} \cdot r_{KL}$ |
|  | Torque of spiral or leg springs | $M_S = k_m \cdot \Delta\varphi$ |
| **Calculating the load torque consisting of component torques** | | |
|  | Addition of torques acting in same direction | $M_L = M_1 + M_2 + ... + M_X$ |
|  | Addition of torques acting in opposite directions | $M_L = M_1 - M_2 - ... - M_X$ |

| Symbol | Name | SI |
|---|---|---|
| $F$ | Force | N |
| $F_{KL}$ | Bearing load, axial / radial | N |
| $J$ | Moment of inertia | kgm$^2$ |
| $M$ | Torque | Nm |
| $M_L$ | Load torque | Nm |
| $M_R$ | Friction torque | Nm |
| $M_S$ | Torque, spiral spring | Nm |
| $M_\alpha$ | Torque for acceleration | Nm |
| $M_1/M_2/M_x$ | Partial torques | Nm |
| $k_m$ | Torsion coefficient (spring constant) | Nm |

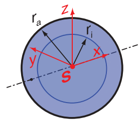
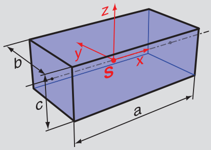
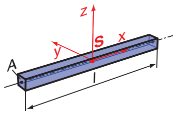
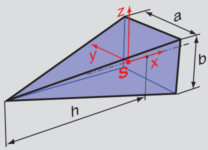
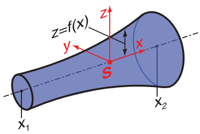
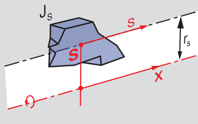
| Symbol | Name | SI |
|---|---|---|
| $r$ | Radius | m |
| $r_{KL}$ | Mean radius bearing | m |
| $\alpha$ | Angular acceleration | rad/s$^2$ |
| $\Delta t$ | Duration | s |
| $\Delta\varphi$ | Rotation angle change | rad |
| $\Delta\omega$ | Angular velocity change | rad/s |
| $\mu$ | Coefficient of friction (see table chapt. 10.2) | |

| Symbol | Name | maxon |
|---|---|---|
| $\Delta n$ | Speed change | rpm |

## 1.3 Moments of inertia of various bodies with reference to the principal axes through the center of gravity S

| Body type | Illustration | Mass, moments of inertia |
|---|---|---|
| **Circular cylinder, disc** |  | $m = \rho \cdot \pi \cdot r^2 \cdot h$<br>$J_x = \frac{1}{2} \cdot m \cdot r^2$<br>$J_y = J_z = \frac{1}{12} \cdot m \cdot (3r^2 + h^2)$ |
| **Hollow cylinder** |  | $m = \rho \cdot \pi \cdot (r_a^2 - r_i^2) \cdot h$<br>$J_x = \frac{1}{2} \cdot m \cdot (r_a^2 + r_i^2)$<br>$J_y = J_z = \frac{1}{4} \cdot m \cdot \left(r_a^2 + r_i^2 + \frac{h^2}{3}\right)$ |
| **Circular cone** |  | $m = \frac{1}{3} \cdot \rho \cdot \pi \cdot r^2 \cdot h$<br>$J_x = \frac{3}{10} \cdot m \cdot r^2$<br>$J_y = J_z = \frac{3}{80} \cdot m \cdot (4r^2 + h^2)$ |
| **Truncated circular cone** |  | $m = \frac{1}{3} \cdot \rho \cdot \pi \cdot (r_2^2 + r_2 r_1 + r_1^2) \cdot h$<br>$J_x = \frac{3}{10} \cdot m \cdot \frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}$ |
| **Circular torus** |  | $m = 2\rho \cdot \pi^2 \cdot r^2 \cdot R$<br>$J_x = J_y = \frac{1}{8} \cdot m \cdot (4R^2 + 5r^2)$<br>$J_z = \frac{1}{4} \cdot m \cdot (4R^2 + 3r^2)$ |
| **Sphere** |  | $m = \frac{4}{3} \cdot \rho \cdot \pi \cdot r^3$<br>$J_x = J_y = J_z = \frac{2}{5} \cdot m \cdot r^2$ |

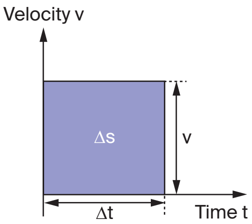
| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $J_x$ | Moment of inertia with reference to the rotation axis *x* | kgm² | $h$ | Height | m |
| | | | $m$ | Mass | kg |
| $J_y$ | Moment of inertia with reference to the rotation axis *y* | kgm² | $r$ | Radius | m |
| | | | $r_a$ | Outer radius | m |
| $J_z$ | Moment of inertia with reference to the rotation axis *z* | kgm² | $r_i$ | Inner radius | m |
| | | | $r_1$ | Radius 1 | m |
| $R$ | Radius circular torus around *z*-axis | m | $r_2$ | Radius 2 | m |
| | | | $\rho$ | Density | kg/m³ |

| Body type | Illustration | Mass, moments of inertia |
|---|---|---|
| **Hollow sphere** |  | $m = \frac{4}{3} \cdot \rho \cdot \pi \cdot (r_a^3 - r_i^3)$<br>$J_x = J_y = J_z = \frac{2}{5} \cdot m \cdot \frac{r_a^5 - r_i^5}{r_a^3 - r_i^3}$ |
| **Cuboid** |  | $m = \rho \cdot a \cdot b \cdot c$<br>$J_x = \frac{1}{12} \cdot m \cdot (b^2 + c^2)$ |
| **Thin rod** |  | $m = \rho \cdot A \cdot l$<br>$J_y = J_z = \frac{1}{12} \cdot m \cdot l^2$ |
| **Square pyramid** |  | $m = \frac{1}{3} \cdot \rho \cdot a \cdot b \cdot h$<br>$J_x = \frac{1}{20} \cdot m \cdot (a^2 + b^2)$<br>$J_y = \frac{1}{20} \cdot m \cdot (b^2 + \frac{3}{4} h^2)$ |
| **Arbitrary rotation body** |  | $m = \rho \cdot \pi \cdot \int_{x1}^{x2} f^2(x) \cdot dx$<br>$J_x = \frac{1}{2} \cdot \rho \cdot \pi \cdot \int_{x1}^{x2} f^4(x) \cdot dx$ |
| **Steiner's theorem**<br>Moment of inertia with reference to a parallel axis of rotation *x* at a distance of $r_s$ to axis *s* through the center of gravity *S*. |  | $J_x = m \cdot r_s^2 + J_s$ |

| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $A$ | Cross section | $m^2$ | $c$ | Length of side *c* | m |
| $J_s$ | Moment of inertia with reference to axis *s* through center of gravity *S* | $kgm^2$ | $h$ | Height | m |
| | | | $l$ | Length | m |
| $J_x$ | Moment of inertia with reference to the rotation axis *x* | $kgm^2$ | $m$ | Mass | kg |
| | | | $r_a$ | Outer radius | m |
| $J_y$ | Moment of inertia with reference to the rotation axis *y* | $kgm^2$ | $r_i$ | Inner radius | m |
| | | | $r_s$ | Distance of axis *s* from center of gravity *S* | m |
| $J_z$ | Moment of inertia with reference to the rotation axis *z* | $kgm^2$ | $\rho$ | Density | $kg/m^3$ |
| $a$ | Length of side *a* | m | $x_1$ | Point 1 on the *x*-axis | m |
| $b$ | Length of side *b* | m | $x_2$ | Point 2 on the *x*-axis | m |

# 2. Kinematics
## 2.1 Linear equations of motion

### Uniform movement

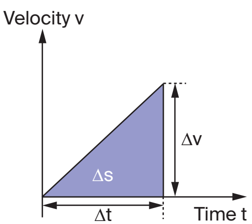



Velocity
$v = \Delta s / \Delta t$ = constant
$[v]$ = m/s

$$v = \frac{\Delta s}{\Delta t}$$

$$\Delta s = v \cdot \Delta t$$

### Constant acceleration from a standing start




Acceleration
$a = \Delta v / \Delta t$ = constant
$[a]$ = m/s²

$$\Delta v = a \cdot \Delta t$$

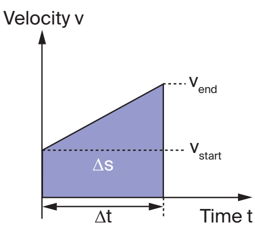
$$\Delta s = \frac{1}{2} \cdot a \cdot \Delta t^2$$

Free fall

$$\Delta v = g \cdot \Delta t$$

$$h = \frac{1}{2} \cdot g \cdot \Delta t^2$$

### Constant acceleration from initial speed




$$v_{end} = v_{start} + a \cdot \Delta t$$

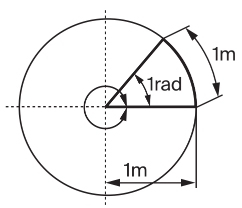
$$\Delta s = v_{start} \cdot \Delta t + \frac{1}{2}\, a \cdot \Delta t^2$$

| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $a$ | Acceleration | m/s² | $t$, $\Delta t$ | Time, duration | s |
| $g$ | Gravitational acceleration | m/s² | $v$, $\Delta v$ | Velocity, velocity change | m/s |
| $h$ | Drop height | m | $v_{end}$ | Velocity after acceleration | m/s |
| $\Delta s$ | Distance change | m | $v_{start}$ | Velocity before acceleration | m/s |

**Remark:**
– The shaded areas represent the distance $\Delta s$ traveled during time period $\Delta t$.

## 2.2 Rotary equations of motion

### General




Conversion between radian and degrees
(The unit *rad* is frequently omitted.)

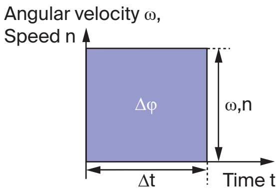
$$\text{rad} = \frac{360°}{2\pi} = 57.2958°$$

$$1° = \frac{2\pi\,\text{rad}}{360} = 0.01745\ \text{rad}$$

Conversion between angular velocity and speed of rotation

$$\omega = \frac{\pi}{30} \cdot n \qquad n = \frac{30}{\pi} \cdot \omega$$

### Uniform movement



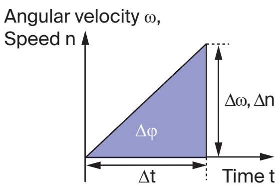

Angular velocity
$\omega = \Delta\varphi / \Delta t$ = constant
$[\omega]$ = rad/s

$$\omega = \frac{\Delta\varphi}{\Delta t}$$

Speed of rotation
$n = 30 / \pi \cdot \Delta\varphi / \Delta t$ = const.
$[n]$ = 1/min = rpm

$$n = \frac{30}{\pi} \cdot \frac{\Delta\varphi}{\Delta t}$$

### Constant acceleration from a standing start




Acceleration
$\alpha = \Delta\omega / \Delta t$ = constant
$[\alpha] = 1/s^2 = rad/s^2$

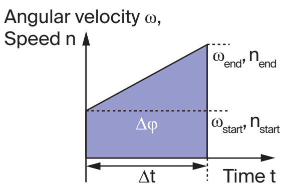
$$\Delta\omega = \alpha \cdot \Delta t$$

$$\Delta n = \frac{30}{\pi} \cdot \alpha \cdot \Delta t$$

$$\Delta\varphi = \frac{1}{2} \cdot \alpha \cdot \Delta t^2$$

$$\Delta\varphi = \frac{1}{2} \cdot \frac{\pi}{30} \cdot \Delta n \cdot \Delta t$$

### Constant acceleration from initial speed




$$\omega_{end} = \omega_{start} + \alpha \cdot \Delta t$$

$$n_{end} = n_{start} + \frac{30}{\pi} \cdot \alpha \cdot \Delta t$$

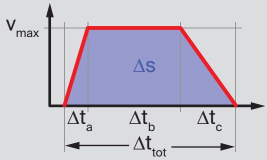
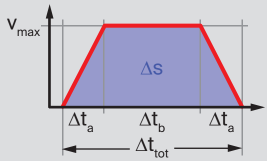
$$\Delta\varphi = \omega_{start} \cdot \Delta t + \frac{1}{2} \cdot \alpha \cdot \Delta t^2$$

$$\Delta\varphi = \frac{\pi}{30} \cdot n_{start} \cdot \Delta t + \frac{1}{2} \cdot \frac{\pi}{30} \cdot \Delta n \cdot \Delta t$$

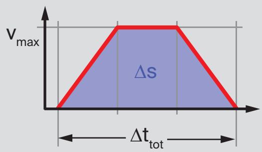
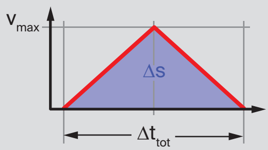
| Symbol | Name | SI | Symbol | Name | maxon |
|---|---|---|---|---|---|
| $t$, $\Delta t$ | Time, duration | s | $n$, $\Delta n$ | Speed of rotation (change) | rpm |
| $\alpha$ | Angular acceleration | rad/s² | $n_{end}$ | Speed after acceleration | rpm |
| $\Delta\varphi$ | Rotation angle change | rad | $n_{start}$ | Speed before acceleration | rpm |
| $\omega$, $\Delta\omega$ | Angular velocity (change) | rad/s | | | |
| $\omega_{end}$ | Angular velocity after acceleration | rad/s | | | |
| $\omega_{start}$ | Angular velocity before acceleration | rad/s | | | |

**Remarks:**
- The shaded areas represent the angle of rotation $\Delta\varphi$ traveled during time period $\Delta t$.
- Angle of rotation $\Delta\varphi = 2\pi$ rad · Number of revolutions = 360° · Number of revolutions

## 2.3 Typical linear motion profiles

| Profile | General | Symmetrical |
|---|---|---|
| **Suitability** | | Travel over long distance at limited velocity |
| **Diagram** |  |  |
| **Task:** | | |
| Travel a distance $\varDelta s$ in time $\varDelta t_{tot}$ | $v_{max} = \dfrac{\varDelta s}{\varDelta t_{tot} - \dfrac{\varDelta t_a + \varDelta t_c}{2}}$<br>$a_{max} = \dfrac{v_{max}}{\varDelta t_a}$ | $v_{max} = \dfrac{\varDelta s}{(\varDelta t_{tot} - \varDelta t_a)}$<br>$a_{max} = \dfrac{\varDelta s}{(\varDelta t_{tot} - \varDelta t_a) \cdot \varDelta t_a}$ |
| Travel a distance $\varDelta s$ at maximum velocity of $v_{max}$ | $\varDelta t_{tot} = \dfrac{\varDelta s}{v_{max}} + \dfrac{\varDelta t_a + \varDelta t_c}{2}$<br>$a_{max} = \dfrac{v_{max}}{\varDelta t_a}$ | $\varDelta t_{tot} = \dfrac{\varDelta s}{v_{max}} + \varDelta t_a$<br>$a_{max} = \dfrac{v_{max}}{\varDelta t_a}$ |
| Travel a distance $\varDelta s$ at maximum acceleration of $a_{max}$ | | $\varDelta t_{tot} = \dfrac{\varDelta s}{a_{max} \cdot \varDelta t_a} + \varDelta t_a$<br>$v_{max} = a_{max} \cdot \varDelta t_a$ |
| Complete motion in the time $\varDelta t_{tot}$ at maximum velocity $v_{max}$ | $\varDelta s = \left(\dfrac{\varDelta t_a + \varDelta t_c}{2} + \varDelta t_b\right) \cdot v_{max}$<br>$a_{max} = \dfrac{v_{max}}{\varDelta t_a}$ | $\varDelta s = (\varDelta t_{tot} - \varDelta t_a) \cdot v_{max}$<br>$a_{max} = \dfrac{v_{max}}{\varDelta t_a}$ |
| Complete motion in the time $\varDelta t_{tot}$ at maximum acceleration $a_{max}$ | | $\varDelta s = a_{max} \cdot (\varDelta t_{tot} - \varDelta t_a) \cdot \varDelta t_a$<br>$v_{max} = a_{max} \cdot \varDelta t_a$ |
| Motion at maximum velocity $v_{max}$ and maximum acceleration $a_{max}$ | | |

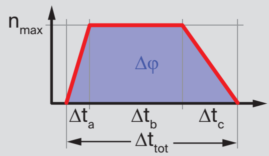
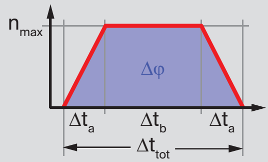
| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $a_{max}$ | Maximum acceleration | m/s² | $\varDelta t_a$ | Time *a* | s |
| $v_{max}$ | Maximum velocity | m/s | $\varDelta t_b$ | Time *b* | s |
| $\varDelta s$ | Distance change | m | $\varDelta t_c$ | Time *c* | s |
| | | | $\varDelta t_{tot}$ | Total time | s |

| 3/3 Trapezoidal | Triangle |
|---|---|
| Optimized for minimum power (at given $\Delta s$ and $\Delta t$): Most advantageous from a thermal point of view | Optimized for limited acceleration or force (at given $\Delta s$ and $\Delta t$). Optimized for minimum time requirement (at given $\Delta s$ and $a_{max}$). |
|   |   |

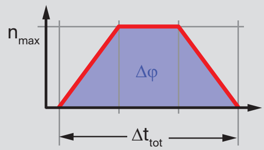
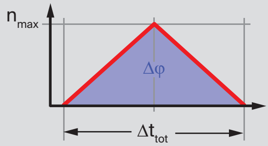
| | |
|---|---|
| $v_{max} = 1.5 \cdot \frac{\Delta s}{\Delta t_{tot}}$<br>$a_{max} = 4.5 \cdot \frac{\Delta s}{\Delta t_{tot}^2}$ | $v_{max} = 2 \cdot \frac{\Delta s}{\Delta t_{tot}}$<br>$a_{max} = 4 \cdot \frac{\Delta s}{\Delta t_{tot}^2}$ |
| $\Delta t_{tot} = 1.5 \cdot \frac{\Delta s}{v_{max}}$<br>$a_{max} = 2 \cdot \frac{v_{max}^2}{\Delta s}$ | $\Delta t_{tot} = 2 \cdot \frac{\Delta s}{v_{max}}$<br>$a_{max} = \frac{v_{max}^2}{\Delta s}$ |
| $\Delta t_{tot} = \frac{3}{\sqrt{2}} \cdot \sqrt{\frac{\Delta s}{a_{max}}} \approx 2.12 \cdot \sqrt{\frac{\Delta s}{a_{max}}}$<br>$v_{max} = \frac{1}{\sqrt{2}} \cdot \sqrt{\Delta s \cdot a_{max}} \approx 0.7 \cdot \sqrt{\Delta s \cdot a_{max}}$ | $\Delta t_{tot} = 2 \cdot \sqrt{\frac{\Delta s}{a_{max}}}$<br>$v_{max} = \sqrt{\Delta s \cdot a_{max}}$ |
| $\Delta s = \frac{2}{3} \cdot \Delta t_{tot} \cdot v_{max}$<br>$a_{max} = 3 \cdot \frac{v_{max}}{\Delta t_{tot}}$ | $\Delta s = \frac{1}{2} \cdot \Delta t_{tot} \cdot v_{max}$<br>$a_{max} = 2 \cdot \frac{v_{max}}{\Delta t_{tot}}$ |
| $\Delta s = \frac{2}{9} \cdot a_{max} \cdot \Delta t_{tot}^2 \approx 0.22 \cdot a_{max} \Delta t_{tot}^2$<br>$v_{max} = \frac{1}{3} \cdot a_{max} \cdot \Delta t_{tot} \approx 0.33 \cdot a_{max} \Delta t_{tot}$ | $\Delta s = \frac{1}{4} \cdot a_{max} \cdot \Delta t_{tot}^2$<br>$v_{max} = \frac{1}{2} \cdot a_{max} \cdot \Delta t_{tot}$ |
| $\Delta s = 2 \cdot \frac{v_{max}^2}{a_{max}}$<br>$\Delta t_{tot} = 3 \cdot \frac{v_{max}}{a_{max}}$ | $\Delta s = \frac{v_{max}^2}{a_{max}}$<br>$\Delta t_{tot} = 2 \cdot \frac{v_{max}}{a_{max}}$ |

| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $a_{max}$ | Maximum acceleration | $m/s^2$ | $\Delta s$ | Distance change | m |
| $v_{max}$ | Maximum velocity | m/s | $\Delta t_{tot}$ | Total time | s |

## 2.4 Typical rotary motion profiles

| Profile | General | Symmetrical |
|---|---|---|
| **Suitability** | | Long rotation at limited speed of rotation |
| **Diagram** |  |  |
| **Task:** | | |
| Travel an angle $\Delta\varphi$ in time $\Delta t_{tot}$ | $n_{max} = \frac{30}{\pi} \cdot \frac{\Delta\varphi}{\Delta t_{tot} - \frac{\Delta t_a + \Delta t_c}{2}}$ <br> $\alpha_{max} = \frac{\Delta\varphi}{\left(\Delta t_{tot} - \frac{\Delta t_a + \Delta t_c}{2}\right) \cdot \Delta t_a}$ | $n_{max} = \frac{30}{\pi} \cdot \frac{\Delta\varphi}{(\Delta t_{tot} - \Delta t_a)}$ <br> $\alpha_{max} = \frac{\Delta\varphi}{(\Delta t_{tot} - \Delta t_a) \cdot \Delta t_a}$ |
| Travel an angle $\Delta\varphi$ at maximum speed $n_{max}$ | $\Delta t_{tot} = \frac{30}{\pi} \cdot \frac{\Delta\varphi}{n_{max}} + \frac{\Delta t_a + \Delta t_c}{2}$ <br> $\alpha_{max} = \frac{\pi}{30} \cdot \frac{n_{max}}{\Delta t_a}$ | $\Delta t_{tot} = \frac{30}{\pi} \cdot \frac{\Delta\varphi}{n_{max}} + \Delta t_a$ <br> $\alpha_{max} = \frac{\pi}{30} \cdot \frac{n_{max}}{\Delta t_a}$ |
| Travel an angle $\Delta\varphi$ at maximum angular acceleration $\alpha_{max}$ | | $\Delta t_{tot} = \frac{\Delta\varphi}{\alpha_{max} \cdot \Delta t_a} + \Delta t_a$ <br> $n_{max} = \frac{30}{\pi} \cdot \alpha_{max} \cdot \Delta t_a$ |
| Complete motion in the time $\Delta t_{tot}$ at maximum speed $n_{max}$ | $\Delta\varphi = \frac{\pi}{30} \cdot n_{max} \cdot \left(\frac{\Delta t_a + \Delta t_c}{2} + \Delta t_b\right)$ <br> $\alpha_{max} = \frac{\pi}{30} \cdot \frac{n_{max}}{\Delta t_a}$ | $\Delta\varphi = \frac{\pi}{30} \cdot n_{max} \cdot (\Delta t_{tot} - \Delta t_a)$ <br> $\alpha_{max} = \frac{\pi}{30} \cdot \frac{n_{max}}{\Delta t_a}$ |
| Complete motion in the time $\Delta t_{tot}$ at maximum angular acceleration $\alpha_{max}$ | | $\Delta\varphi = \alpha_{max} \cdot (\Delta t_{tot} - \Delta t_a) \cdot \Delta t_a$ <br> $n_{max} = \frac{30}{\pi} \cdot \alpha_{max} \cdot \Delta t_a$ |
| Motion at maximum speed $n_{max}$ and maximum angular acceleration $\alpha_{max}$ | | |

| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $\alpha_{max}$ | Maximum angular acceleration | rad/s² | $\Delta\varphi$ | Rotation angle change | rad |
| $\Delta t_a$ | Time *a* | s | | | |
| $\Delta t_b$ | Time *b* | s | **Symbol** | **Name** | **maxon** |
| $\Delta t_c$ | Time *c* | s | $n_{max}$ | Maximum speed in load cycle | rpm |
| $\Delta t_{tot}$ | Total time | s | | | |

| 3/3 Trapezoidal | Triangle |
|---|---|
| Optimized for minimum power (at given $\Delta\varphi$ and $\Delta t$): Most advantageous from a thermal point of view | Optimized for limited angular acceleration or torque (at given $\Delta\varphi$ and $\Delta t$) and for minimum time requirement (at given $\Delta\varphi$ and $\alpha_{max}$). |
| <br> | <br> |
| $n_{max} = 1.5 \cdot \frac{30}{\pi} \cdot \frac{\Delta\varphi}{\Delta t_{tot}}$<br>$\alpha_{max} = 4.5 \cdot \frac{\Delta\varphi}{\Delta t_{tot}^{2}}$ | $n_{max} = 2 \cdot \frac{30}{\pi} \cdot \frac{\Delta\varphi}{\Delta t_{tot}}$<br>$\alpha_{max} = 4 \cdot \frac{\Delta\varphi}{\Delta t_{tot}^{2}}$ |
| $\Delta t_{tot} = 1.5 \cdot \frac{30}{\pi} \cdot \frac{\Delta\varphi}{n_{max}}$<br>$\alpha_{max} = 2 \cdot \frac{\pi^2}{30^2} \cdot \frac{n_{max}^{2}}{\Delta\varphi}$ | $\Delta t_{tot} = 2 \cdot \frac{30}{\pi} \cdot \frac{\Delta\varphi}{n_{max}}$<br>$\alpha_{max} = \frac{\pi^2}{30^2} \cdot \frac{n_{max}^{2}}{\Delta\varphi}$ |
| $\Delta t_{tot} = \frac{3}{\sqrt{2}} \cdot \sqrt{\frac{\Delta\varphi}{\alpha_{max}}} \approx 2.12 \cdot \sqrt{\frac{\Delta\varphi}{\alpha_{max}}}$<br>$n_{max} = \frac{1}{\sqrt{2}} \cdot \frac{30}{\pi} \cdot \sqrt{\Delta\varphi \cdot \alpha_{max}} \approx 6.75 \cdot \sqrt{\Delta\varphi \cdot \alpha_{max}}$ | $\Delta t_{tot} = 2 \cdot \sqrt{\frac{\Delta\varphi}{\alpha_{max}}}$<br>$n_{max} = \frac{30}{\pi} \sqrt{\Delta\varphi \cdot \alpha_{max}}$ |
| $\Delta\varphi = \frac{2}{3} \cdot \frac{\pi}{30} \cdot \Delta t_{tot} \cdot n_{max}$<br>$\alpha_{max} = 3 \cdot \frac{\pi}{30} \cdot \frac{n_{max}}{\Delta t_{tot}}$ | $\Delta\varphi = \frac{1}{2} \cdot \frac{\pi}{30} \cdot \Delta t_{tot} \cdot n_{max}$<br>$\alpha_{max} = 2 \cdot \frac{\pi}{30} \cdot \frac{n_{max}}{\Delta t_{tot}}$ |
| $\Delta\varphi = \frac{2}{9} \cdot \alpha_{max} \cdot \Delta t_{tot}^{2} \approx 0.22 \cdot \alpha_{max} \cdot \Delta t_{tot}^{2}$<br>$n_{max} = \frac{1}{3} \cdot \frac{30}{\pi} \cdot \alpha_{max} \cdot \Delta t_{tot} \approx 3.18 \cdot \alpha_{max} \cdot \Delta t_{tot}$ | $\Delta\varphi = \frac{1}{4} \cdot \alpha_{max} \cdot \Delta t_{tot}^{2}$<br>$n_{max} = \frac{1}{2} \cdot \frac{30}{\pi} \cdot \alpha_{max} \cdot \Delta t_{tot}$ |
| $\Delta\varphi = 2 \cdot \frac{30}{\pi} \cdot \frac{n_{max}^{2}}{\alpha_{max}}$<br>$\Delta t_{tot} = 3 \cdot \frac{\pi}{30} \cdot \frac{n_{max}}{\alpha_{max}}$ | $\Delta\varphi = \frac{30}{\pi} \cdot \frac{n_{max}^{2}}{\alpha_{max}}$<br>$\Delta t_{tot} = 2 \cdot \frac{\pi}{30} \cdot \frac{n_{max}}{\alpha_{max}}$ |

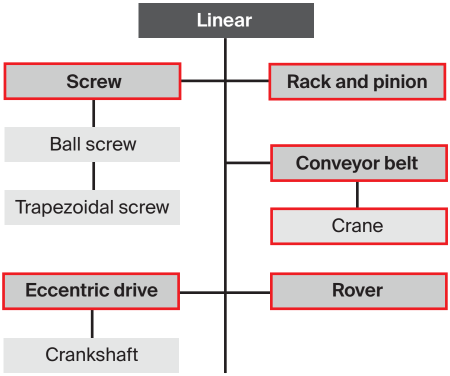
| Symbol | Name | SI |
|---|---|---|
| $\alpha_{max}$ | Maximum angular acceleration | rad/s² |
| $\Delta t_{tot}$ | Total time | s |
| $\Delta\varphi$ | Rotation angle change | rad |

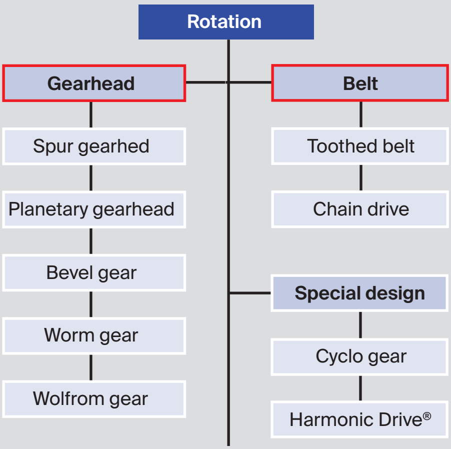
| Symbol | Name | maxon |
|---|---|---|
| $n_{max}$ | Maximum speed in load cycle | rpm |

# Notes

# 3. Mechanical drives
## 3.1 Mechanical transmission

### Classification




### Output power / transmission, general

**Output power, linear motion**

$[W] = \mathrm{m/s} \cdot \mathrm{N}$

$$P_{mech} = v \cdot F$$

**Transmission, general**

$$P_{L,mech} = \frac{P_{in,mech}}{\eta}$$

$$v_L \cdot F_L = \frac{\omega_{in} \cdot M_{in}}{\eta}$$

**Output power, rotary motion**

$[W] = \mathrm{s}^{-1} \cdot \mathrm{Nm}$

$$P_{mech} = \omega \cdot M = \frac{\pi}{30} \cdot n \cdot M$$

**Transmission, general**

$$P_{L,mech} = \frac{P_{in,mech}}{\eta}$$

$$\omega_L \cdot M_L = \frac{\omega_{in} \cdot M_{in}}{\eta}$$

### Designations in the formulae

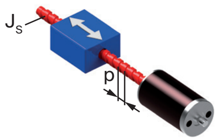
- The load-side variables at the output are identified by the index *L*.
- The input-side variables (usually the motor) are identified by the index *in*.

| Symbol | Name | SI |
|---|---|---|
| $F$ | Force | N |
| $F_L$ | Load force (output) | N |
| $M$ | Torque | Nm |
| $M_{in}$ | Input torque | Nm |
| $M_L$ | Load torque | Nm |
| $P_{in,mech}$ | Mechanical input power | W |
| $P_{L,mech}$ | Mechanical output power | W |
| $P_{mech}$ | Mechanical power | W |
| $v$ | Velocity | m/s |

| Symbol | Name | SI |
|---|---|---|
| $v_L$ | Load velocity | m/s |
| $\eta$ | Efficiency | |
| $\omega$ | Angular velocity | rad/s |
| $\omega_L$ | Angular velocity load | rad/s |
| $\omega_{in}$ | Angular velocity input | rad/s |

| Symbol | Name | maxon |
|---|---|---|
| $n$ | Speed of rotation | rpm |

## Screw drive



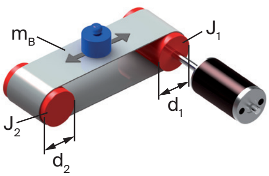

| | |
|---|---|
| Speed of rotation | $n_{in} = \frac{60}{p} \cdot v_L$ |
| Torque | $M_{in} = \frac{p}{2\pi} \cdot \frac{F_L}{\eta}$ |

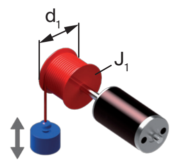
Additional torque for constant acceleration (speed change $\Delta n_{in}$ during period $\Delta t_a$)

$$M_{in,\alpha} = \left( J_{in} + J_S + \frac{m_L + m_S}{\eta} \cdot \frac{p^2}{4\pi^2} \right) \cdot \frac{\pi}{30} \cdot \frac{\Delta n_{in}}{\Delta t_a}$$

| | |
|---|---|
| Play, position error | $\Delta\varphi_{in} = \Delta s_L \cdot \frac{2\pi}{p}$ |

## Belt drive / conveyor belt / crane




| | |
|---|---|
| Speed of rotation | $n_{in} = \frac{60}{\pi} \cdot \frac{v_L}{d_1}$ (Assumption: no slip) |
| Torque | $M_{in} = \frac{d_1}{2} \cdot \frac{F_L}{\eta}$ |

Additional torque for constant acceleration (speed change $\Delta n_{in}$ during period $\Delta t_a$)

$$M_{in,\alpha} = \left( J_{in} + J_1 + \frac{J_2}{\eta} \cdot \frac{d_1^2}{d_2^2} + \frac{J_X}{\eta} \cdot \frac{d_1^2}{d_X^2} + \frac{m_L + m_B}{\eta} \cdot \frac{d_1^2}{4} \right) \cdot \frac{\pi}{30} \cdot \frac{\Delta n_{in}}{\Delta t_a}$$

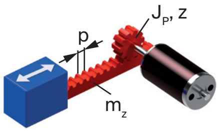
| | |
|---|---|
| Play, position error | $\Delta\varphi_{in} = \Delta s_L \cdot \frac{2}{d_1}$ |

| Symbol | Name | SI |
|---|---|---|
| $F_L$ | Load force (output) | N |
| $J_{in}$ | Moment of inertia, input (motor, encoder, brake) | kgm² |
| $J_S$ | Moment of inertia, screw | kgm² |
| $J_X$ | Moment of inertia, deflector pulley *X* | kgm² |
| $J_1$ | Moment of inertia, driving end | kgm² |
| $J_2$ | Moment of inertia, deflector pulley 2 | kgm² |
| $M_{in}$ | Input torque | Nm |
| $M_{in,\alpha}$ | Torque for acceleration | Nm |
| $d_X$ | Diameter, deflector pulley *X* | m |
| $d_1$ | Diameter, drive pulley | m |
| $d_2$ | Diameter, deflector pulley 2 | m |
| $m_B$ | Mass, belt | kg |

| Symbol | Name | SI |
|---|---|---|
| $m_L$ | Mass of the load | kg |
| $m_S$ | Mass, screw nut | kg |
| $p$ | Screw lead (pitch) | m |
| $v_L$ | Load velocity | m/s |
| $\Delta s_L$ | Mechanical play, load | m |
| $\Delta t_a$ | Acceleration time | s |
| $\Delta\varphi_{in}$ | Mechanical play, input | rad |
| $\eta$ | Efficiency | |

| Symbol | Name | maxon |
|---|---|---|
| $n_{in}$ | Input speed | rpm |
| $\Delta n_{in}$ | Speed change, input | rpm |

## Rack-and-pinion drive



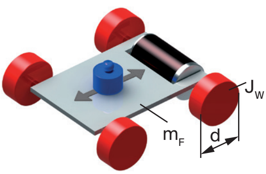

| | |
|---|---|
| Speed of rotation | $n_{in} = \frac{60}{p \cdot z} \cdot v_L$ |
| Torque | $M_{in} = \frac{p \cdot z}{2\pi} \cdot \frac{F_L}{\eta}$ |

Additional torque for constant acceleration (speed change $\Delta n_{in}$ during period $\Delta t_a$)

$$M_{in,\alpha} = \left( J_{in} + J_P + \frac{m_L + m_Z}{\eta} \cdot \frac{p^2 \cdot z^2}{4\pi^2} \right) \cdot \frac{\pi}{30} \cdot \frac{\Delta n_{in}}{\Delta t_a}$$

| | |
|---|---|
| Play, position error | $\Delta\varphi_{in} = \Delta s_L \cdot \frac{2\pi}{p \cdot z}$ |

## Rover




| | |
|---|---|
| Speed of rotation | $n_{in} = \frac{60}{\pi} \cdot \frac{v_L}{d}$ (Assumption: no slip) |
| Torque | $M_{in} = \frac{d}{2} \cdot \frac{F_L}{\eta}$ |

Additional torque for constant acceleration (speed change $\Delta n_{in}$ during period $\Delta t_a$)

$$M_{in,\alpha} = \left( J_{in} + J_W + \frac{m_L + m_F}{\eta} \cdot \frac{d^2}{4} \right) \cdot \frac{\pi}{30} \cdot \frac{\Delta n_{in}}{\Delta t_a}$$

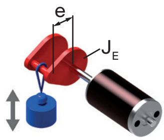
| | |
|---|---|
| Play, position error | $\Delta\varphi_{in} = \Delta s_L \cdot \frac{2}{d}$ |

| Symbol | Name | SI |
|---|---|---|
| $F_L$ | Load force (output) | N |
| $J_{in}$ | Moment of inertia, input (motor, encoder, brake) | kgm² |
| $J_P$ | Moment of inertia, pinion | kgm² |
| $J_W$ | Moment of inertia, all wheels together | kgm² |
| $M_{in}$ | Input torque | Nm |
| $M_{in,\alpha}$ | Torque for acceleration | Nm |
| $d$ | Diameter, drive wheel | m |
| $m_F$ | Mass, rover | kg |
| $m_L$ | Mass of the load | kg |

| Symbol | Name | SI |
|---|---|---|
| $m_Z$ | Mass, gear rack | kg |
| $p$ | Pitch | m |
| $v_L$ | Load velocity | m/s |
| $z$ | Number of teeth, pinion | |
| $\Delta s_L$ | Mechanical play, load | m |
| $\Delta t_a$ | Acceleration time | s |
| $\Delta\varphi_{in}$ | Mechanical play, input | rad |
| $\eta$ | Efficiency | |

| Symbol | Name | maxon |
|---|---|---|
| $n_{in}$ | Input speed | rpm |
| $\Delta n_{in}$ | Speed change, input | rpm |

## Eccentric drive




Sinusoidal velocity curve of the load
(assumption: constant input speed $n_{in}$)

$$v_L(t) = \frac{\pi}{30} \cdot n_{in} \cdot e \cdot sin\left(\frac{\pi}{30} \cdot n_{in} \cdot t\right)$$

Angle-dependent periodic acceleration force for load, pistons and rods ($m_L$)

$$F_a(\varphi) = F_a \cdot cos\,\varphi = m_L \cdot \left(\frac{\pi}{30} \cdot n_{in}\right)^2 \cdot e \cdot cos\,\varphi$$

Angle-dependent torques due to different load conditions in the two half cycles of the back and forth motion.

$$M_{in1}(\varphi) = e \cdot (F_{L1} \cdot sin\,\varphi + F_{a1} \cdot cos\,\varphi) \qquad 0 \leq \varphi \leq \pi$$
$$M_{in2}(\varphi) = e \cdot (F_{L2} \cdot sin\,\varphi + F_{a2} \cdot cos\,\varphi) \qquad \pi \leq \varphi \leq 2\pi$$

Average effective torque load

$$M_{in,RMS} = \frac{e}{\sqrt{2 \cdot \eta}} \cdot \sqrt{F_{L1}^2 + F_{a1}^2 + F_{L2}^2 + F_{a2}^2}$$

Additional torque for acceleration of the eccentric disc
(speed change $\Delta n_{in}$ during period $\Delta t_a$)

$$M_{in,\alpha} = \left(J_{in} + J_E\right) \cdot \frac{\pi}{30} \cdot \frac{\Delta n_{in}}{\Delta t_a}$$

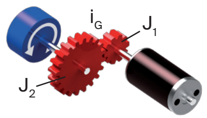
| Symbol | Name | SI |
|---|---|---|
| $F_{L1}$ | Load force 1st half cycle | N |
| $F_{L2}$ | Load force 2nd half cycle | N |
| $F_a$ | Acceleration force | N |
| $F_a(\varphi)$ | Periodic acceleration force as a function of the angle of rotation | N |
| $F_{a1}$ | Acceleration force, 1st / 2nd half cycle | N |
| $F_{a2}$ | Acceleration force, 1st / 2nd half cycle | N |
| $J_{in}$ | Moment of inertia, input (motor, encoder, brake) | $kgm^2$ |
| $J_E$ | Moment of inertia, eccentric disc | $kgm^2$ |
| $M_{in,RMS}$ | RMS torque | Nm |
| $M_{in,\alpha}$ | Torque for acceleration | Nm |
| $M_{in1}(\varphi)$ | Torque, 1st half cycle | Nm |
| $M_{in2}(\varphi)$ | Torque, 2nd half cycle | Nm |

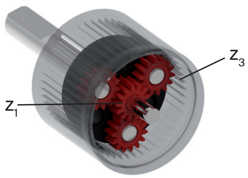
| Symbol | Name | SI |
|---|---|---|
| $e$ | Eccentricity | m |
| $m_L$ | Mass of the load | kg |
| $v_L(t)$ | Sinusoidal velocity curve of the load | m/s |
| $t$ | Time | s |
| $\Delta t_a$ | Acceleration time | s |
| $\varphi$ | Rotation angle | rad |
| $\eta$ | Efficiency | |

| Symbol | Name | maxon |
|---|---|---|
| $n_{in}$ | Input speed | rpm |
| $\Delta n_{in}$ | Speed change, input | rpm |

## 3.3 Mechanical drives, rotation → rotation

### Gearhead



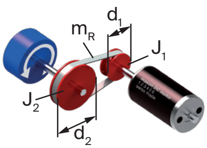

| | |
|---|---|
| Speed of rotation | $n_{in} = n_L \cdot i_G$ |
| Torque | $M_{in} = \dfrac{M_L}{i_G \cdot \eta}$ |

Additional torque for constant acceleration (speed change $\Delta n_{in}$ during period $\Delta t_a$)

$$M_{in,\alpha} = \left( J_{in} + J_1 + \frac{J_L + J_2}{i_G^2 \cdot \eta} \right) \cdot \frac{\pi}{30} \cdot \frac{\Delta n_{in}}{\Delta t_a} = \left( J_{in} + J_G + \frac{J_L}{i_G^2 \cdot \eta} \right) \cdot \frac{\pi}{30} \cdot \frac{\Delta n_{in}}{\Delta t_a}$$

| | |
|---|---|
| Play, position error | $\Delta\varphi_{in} = \Delta\varphi_L \cdot i_G$ |
| Reduction ratio planetary gearhead | $i_G = \dfrac{z_1 + z_3}{z_1}$ |

### Belt drive




| | |
|---|---|
| Speed of rotation | $n_{in} = n_L \cdot \dfrac{d_2}{d_1}$ (Assumption: no slip) |
| Torque | $M_{in} = \dfrac{d_1}{d_2} \cdot \dfrac{M_L}{\eta}$ |

Additional torque for constant acceleration (speed change $\Delta n_{in}$ during period $\Delta t_a$)

$$M_{in,\alpha} = \left( J_{in} + J_1 + \frac{J_L + J_2}{\eta} \cdot \frac{d_1^2}{d_2^2} + \frac{J_x}{\eta} \cdot \frac{d_1^2}{d_X^2} + \frac{m_R \cdot d_1^2}{4 \cdot \eta} \right) \cdot \frac{\pi}{30} \cdot \frac{\Delta n_{in}}{\Delta t_a}$$

| | |
|---|---|
| Play, position error | $\Delta\varphi_{in} = \Delta\varphi_{out} \cdot \dfrac{d_2}{d_1}$ |

| Symbol | Name | SI |
|---|---|---|
| $J_G$ | Moment of inertia, gearhead transformed | kgm² |
| $J_{in}$ | Moment of inertia, input (motor, encoder, brake) | kgm² |
| $J_L$ | Moment of inertia, load | kgm² |
| $J_x$ | Moment of inertia, deflector pulley *X* | kgm² |
| $J_1$ | Moment of inertia, driving end | kgm² |
| $J_2$ | Moment of inertia, load | kgm² |
| $M_{in}$ | Input torque | Nm |
| $M_{in,\alpha}$ | Torque for acceleration | Nm |
| $M_L$ | Load torque | Nm |
| $d_X$ | Diameter, deflector pulley *X* | m |
| $d_1$ | Diameter, drive pulley | m |
| $d_2$ | Diameter, load pulley | m |

| Symbol | Name | SI |
|---|---|---|
| $i_G$ | Reduction ratio, gearhead (catalog value) | |
| $m_R$ | Mass, belt | kg |
| $z_1$ | Number of teeth, sun wheel | |
| $z_3$ | Number of teeth, internal gear | |
| $\Delta t_a$ | Acceleration time | s |
| $\Delta\varphi_{in}$ | Mechanical play, input | rad |
| $\Delta\varphi_L$ | Mechanical play, load | rad |
| $\eta$ | Efficiency | |

| Symbol | Name | maxon |
|---|---|---|
| $n_{in}$ | Input speed | rpm |
| $n_L$ | Load speed | rpm |
| $\Delta n_{in}$ | Speed change, input | rpm |

## Identification system for maxon-gearheads

**Gearhead design type**

| | |
|---|---|
| **GS** | Spur gearhead |
| **GP** | Planetary gearhead |
| **GPX** | Configurable planetary gearhead |
| **KD** | Koaxdrive |

**GP** **22** **A**

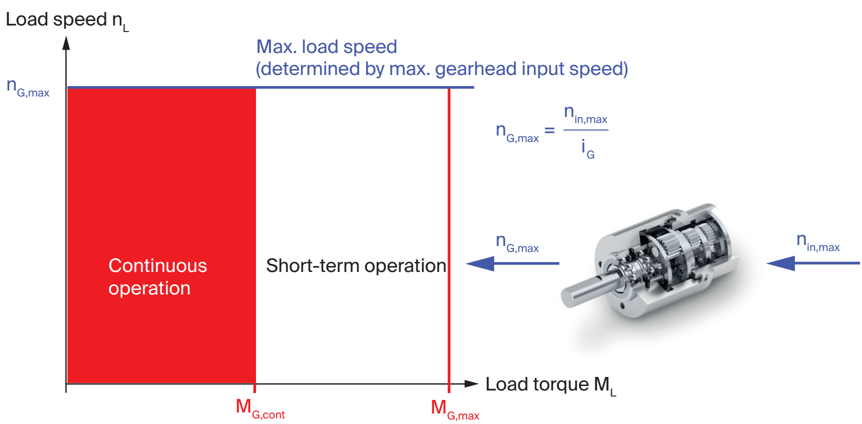
**Diameter** in mm

**Version**

| | |
|---|---|
| **A** | Metal version |
| **AR, CR** | High Radial loads |
| **B** | Version with enlarged internal gear |
| **C** | Ceramic version |
| **HD** | Heavy Duty – for applications in oil |
| **HP** | High Power version |
| **UP** | Ultra high Performance |
| **K** | Plastic version |
| **LN** | Low Noise version |
| **SPEED** | High input speeds |
| **S** | Screw drive with axial bearing |
| **V** | Reinforced version |
| **_Z, LZ** | Low backlash version |

## Operating ranges of gearheads

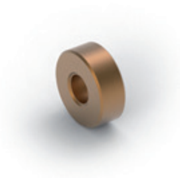
maxon-gearheads are designed for an operating life of at least 1000 hours at the given maximum continuous torque and maximum input speed ratings. Operation below these limits will significantly increase operating life. If the limits are exceeded, the useful life of the gearhead may be reduced.




| Symbol | Name | SI |
|---|---|---|
| $M_{G,cont}$ | Max. continuous torque, gearhead (catalog value) | Nm |
| $M_{G,max}$ | Max. intermittent torque, gearhead (catalog value) | Nm |
| $i_G$ | Reduction ratio, gearhead (catalog value) | |

| Symbol | Name | maxon |
|---|---|---|
| $n_{G,max}$ | Maximum output speed gearhead | rpm |
| $n_{in,max}$ | Maximum input speed (catalog value) | rpm |

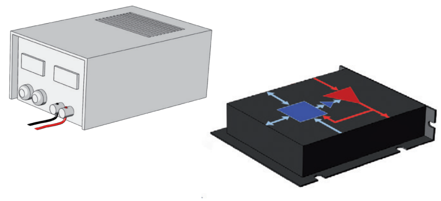
# 4. Bearing
## 4.1 Comparison of characteristics of sintered sleeve bearings and ball bearings

| | Sintered sleeve bearings | Ball bearings |
|---|---|---|
| |  |  |
| **Operating modes** | – continuous operation | – suitable for all types of operations<br>– especially for start-stop operations and low-speed applications |
| **Speed range** | – ideal above approx. 500 rpm (range for hydrodynamic lubrication)<br>– with special material pairing and lubrication even at lower speeds | – up to several 10 000 rpm<br>– in special cases up to 100 000 rpm and higher (e.g. with ceramic balls) |
| **Radial / axial load** | – only small bearing loads | – higher loads<br>– preloaded ball bearings permit axial loads up to the value of the preload. |
| **Additional operating criteria** | – typical in small DC motors up to approx. 30 mm diameter and in spur gearheads<br>– not suitable for rotating load<br>– not suitable for vacuum applications (outgassing)<br>– not suitable for low temperatures (< -20°C) | – typical in DC motors above 10 mm diameter and in planetary gearheads<br>– preloaded ball bearings typical in brushless DC motors guaranteeing a very long life and smooth operation |
| **Bearing play** | – axial: typically 0.05 … 0.15 mm<br>– radial: typically 0.014 mm | – axial: typically 0.05 … 0.15 mm (no axial play if preloaded)<br>– radial: typically 0.025 mm |
| **Coefficient of friction, typical** | – 0.001 … 0.01 (hydrodynamic lubrication) | – 0.001 … 0.1 |
| **Lubrication** | – hydrodynamic lubrication only at high speeds<br>– shaft bearing material very important, pore size of the sintered bearing and viscosity of the lubricant at operating temperature are critical<br>– special: sintered iron bearings with ceramic shaft for high radial loads and long operating life | – temperature range for standard lubrication: typically -20 … 100 °C<br>– special lubrication possible for very high or very low operating temperatures<br>– sealing possible (but higher friction, shorter life and lower speed limit) |
| **Costs** | economical | more expensive |

# Notes

# 5. Electrical principles
## 5.1 Principles of DC (Direct Current)

### Electric power



Power:
$[P] = V \cdot A = VA$
$= W = J/s$

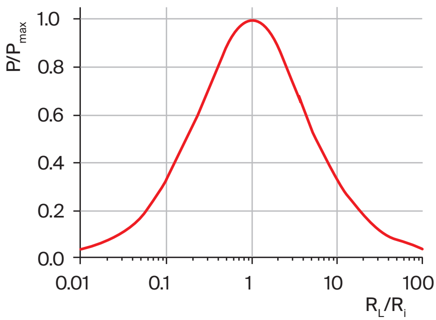
$$P = U \cdot I = R \cdot I^2 = \frac{U^2}{R}$$

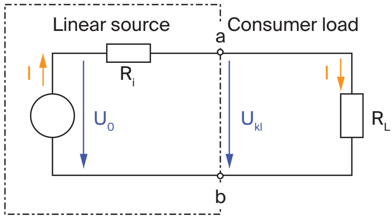
Power loss:

$$P_V = R \cdot I^2$$

### Power adjustment

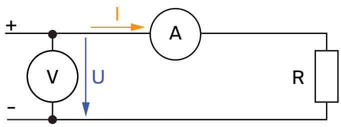
At $R_L = R_i$ the maximum power is drawn from a voltage source.




$$P_{max} = \frac{U_0^2}{4 \cdot R_i} = I^2 \cdot R_i$$

### Ohm's law



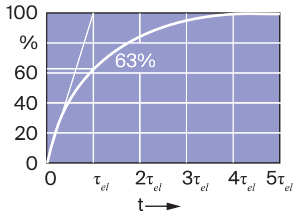

$$U = R \cdot I$$

$$I = \frac{U}{R}$$

$$R = \frac{U}{I}$$

| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $I$ | Current | A | $R_i$ | Inner resistance, voltage source | Ω |
| $P$ | Power | W | $R_L$ | Load resistance | Ω |
| $P_{max}$ | Maximum power | W | $U$ | Voltage | V |
| $P_V$ | Power losses | W | $U_0$ | Source voltage | V |
| $R$ | Electrical resistance | Ω | $U_{kl}$ | Terminal voltage | V |

## Electrical time constant




The electrical time constant describes the reaction time of the current when switching on or off a voltage.

$[\tau_{el}]$ = H/Ω = Vs/A/Ω = s
$[\tau_{el}]$ = Ω · F = Ω · As/V = s

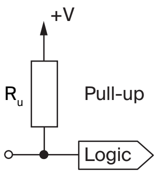
Current change with inductive load

$$\tau_{el} = \frac{L}{R}$$

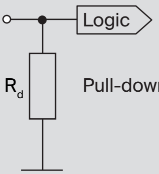
Voltage change with capacitive load

$$\tau_{el} = R \cdot C$$

## Pull-up / pull-down

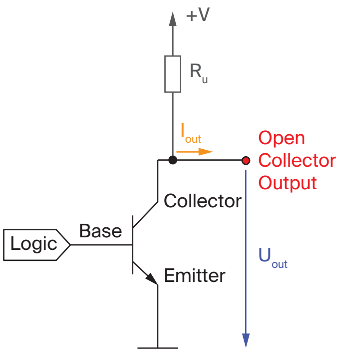



**Pull-up:** (relatively high-impedance) resistor
- Connects signal line with higher voltage potential
- Pulls the line up to the higher potential, if no external voltage actively pulls the line to a lower potential




**Pull-down:** (relatively high-impedance) resistor
- Connects signal line with lower voltage potential
- Pulls the line down to the lower potential, if no external voltage actively pulls the line to a higher potential

## Open-collector output




**Open-collector output (OC):**

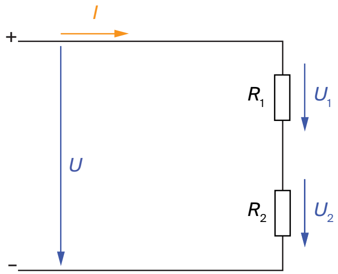
- Output of an integrated circuit with a bipolar transistor with an open collector output.
- Usually the outputs are used in combination with a pull-up resistor which raises the output voltage to a higher potential in the inactive state.

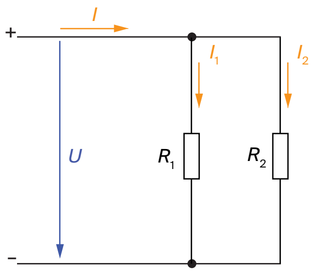
$$U_{out} = +V - (I_{out} \cdot R_u)$$

**Hall sensors** usually have an open-collector output without pull-up resistor. Therefore, it is integrated into the maxon controllers.

| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $C$ | Capacitance | F | $t$ | Time | s |
| $I_{out}$ | Output current | A | $U_{in}$ | Input voltage | V |
| $L$ | Inductance | H | $U_{out}$ | Output voltage | V |
| $R$ | Electrical resistance | Ω | $+V$ | Supply voltage | V |
| $R_d$ | Pull-down resistance | Ω | $\tau_{el}$ | Electrical time constant | s |
| $R_u$ | Pull-up resistance | Ω | | | |

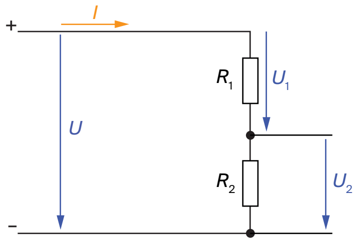
## 5.2 Electrical resistive circuits

| Series resistor circuits | | |
|---|---|---|
|   | I = constant | $U = U_1 + U_2 + ...$ <br> $R = R_1 + R_2 + ...$ <br> $\frac{U_1}{U_2} = \frac{R_1}{R_2}$ |

| Parallel resistor circuits | | |
|---|---|---|
|   | U = constant | $I = I_1 + I_2 + ...$ <br> $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ...$ <br> $R = \frac{R_1 \cdot R_2}{R_1 + R_2}$ <br> $\frac{I_1}{I_2} = \frac{R_2}{R_1}$ |

| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $I$ | Total current | A | $R_1$, $R_2$ | Partial resistances | Ω |
| $I_1$, $I_2$ | Partial currents | A | $U$ | Total voltage | V |
| $R$ | Equivalent resistance | Ω | $U_1$, $U_2$ | Partial voltages | V |

## Voltage divider, no-load

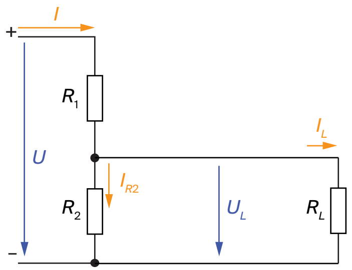


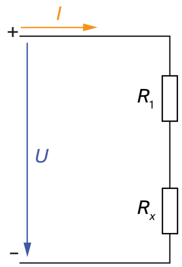

$$U_2 = U \cdot \frac{R_2}{R_1 + R_2}$$

$$\frac{U_1}{U_2} = \frac{R_1}{R_2}$$

$$I = \frac{U_2}{R_2} = \frac{U}{R_1 + R_2}$$

## Voltage divider, under load



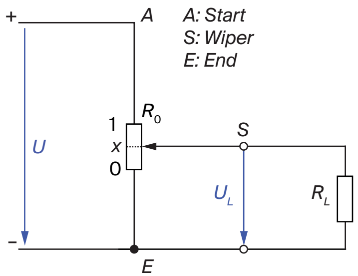

$$U_L = U \cdot \frac{R_x}{R_x + R_1}$$

$$R_x = \frac{R_L \cdot R_2}{R_L + R_2}$$

$$I_L = I \cdot \frac{R_2}{R_L + R_2}$$

$$I_{R2} = I \cdot \frac{R_L}{R_L + R_2}$$

## Potentiometer




| | |
|---|---|
| | $R = x \cdot R_0$ |
| No-load | $U_L = x \cdot U$ |
| Under load | $U_L = U \cdot \dfrac{x}{\left(\dfrac{R_0}{R_L}(x - x^2)\right) + 1}$ |

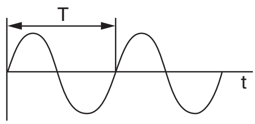
## Winding resistance

| | |
|---|---|
| Temperature-dependence | $R_{TW} = R_{mot} \cdot (1 + \alpha_{Cu} \cdot (T_W - 25\ \text{C°}))$ |

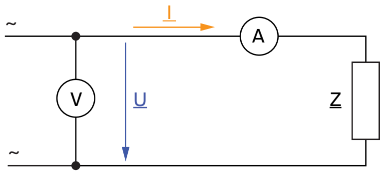
| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $I$ | Total current | A | $R_{TW}$ | Winding resistance at current temp. $T_W$ | Ω |
| $I_L$ | Load current | A | $U$ | Total voltage | V |
| $I_{R2}$ | Current through resistor $R_2$ | A | $U_1$, $U_2$ | Partial voltages | V |
| $R$ | Equivalent resistance | Ω | $U_L$ | Load voltage | V |
| $R_0$ | Resistance, potentiometer | Ω | $T_W$ | Winding temperature | °C |
| $R_1$, $R_2$ | Partial resistances | Ω | $x$ | Potentiometer position | 0…1 |
| $R_L$ | Load resistance | Ω | | | |
| $R_{mot}$ | Terminal resistance, motor (catalog value) | Ω | **Symbol** | **Name** | **Value** |
| $R_x$ | Equivalent resistance of $R_2$ and $R_L$ | Ω | $\alpha_{Cu}$ | Resistance coefficient, copper | 0.0039 $K^{-1}$ |

## 5.3 Principles of AC (Alternating Current)

### Alternating quantities

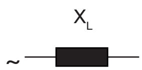
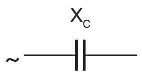
| | | |
|---|---|---|
|   | Frequency<br>$[f] = 1/\mathrm{s} = \mathrm{Hz}$ | $f = \frac{1}{T}$ |
| | Angular frequency<br>$[\omega] = 1/\mathrm{s} = \mathrm{rad/s}$ | $\omega = 2\pi \cdot f$ |

### Ohm's law

| | |
|---|---|
|   | $\underline{U} = \underline{Z} \cdot \underline{I}$<br>$\underline{I} = \frac{\underline{U}}{\underline{Z}}$<br>$\underline{Z} = \frac{\underline{U}}{\underline{I}}$ |

### Resistances

#### Reactance

| | | |
|---|---|---|
|   | Inductive: | $X_L = \omega \cdot L = 2\pi \cdot f \cdot L$ |
|   | Capacitive: | $X_C = \frac{1}{\omega \cdot C} = \frac{1}{2\pi \cdot f \cdot C}$ |

#### Impedance (AC resistance) Z = |$\underline{Z}$|

| | |
|---|---|
| For series connection of *R* and *L*, or *R* and *C* | $Z = \sqrt{R^2 + X^2}$ |

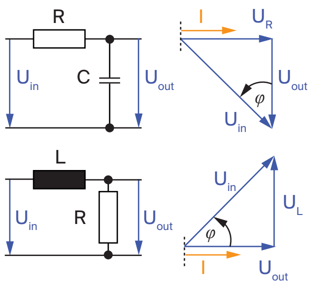
| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $C$ | Capacitance | F | $X$ | Stands for $X_C$ or $X_L$ | Ω |
| $\underline{I}$ | Current | A | $X_C$ | Reactance, capacitive | Ω |
| $L$ | Inductance | H | $X_L$ | Reactance, inductive | Ω |
| $R$ | Electrical resistance | Ω | $\underline{Z}$ | Impedance | Ω |
| $T$ | Period | s | $f$ | Frequency | Hz |
| $\underline{U}$ | Voltage | V | $t$ | Time | s |
| | | | $\omega$ | Angular frequency | rad/s |

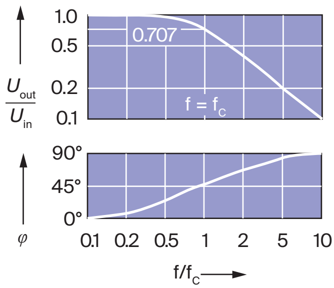
## 5.4 Simple filters

| General | |
|---|---|
| Cut-off frequency $f_C$ | $f_C = \dfrac{1}{2\pi \cdot R \cdot C}$ or $f_C = \dfrac{R}{2\pi \cdot L}$ |
| Phase shift | $\cos\varphi = \dfrac{U_{out}}{U_{in}}$ |

### Low-pass filters, integral element

Allow frequencies to pass virtually unaffected below their cut-off frequency $f_C$.
Higher frequencies are dampened.
Applications: maxon controller inputs, commutation signal measurement of maxon motors.



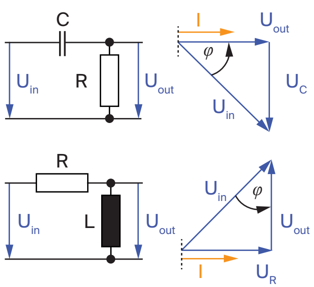

$$\frac{U_{out}}{U_{in}} = \frac{1}{\sqrt{1 + (f/f_C)^2}}$$

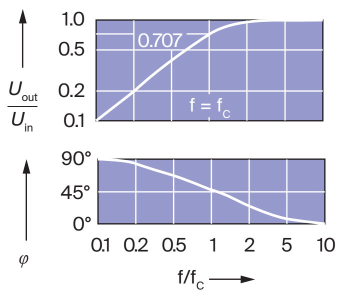
$$U_{out} = U_{in} \frac{X_c}{\sqrt{R^2 + X_c^2}}$$

$$U_{out} = U_{in} \frac{R}{\sqrt{R^2 + X_L^2}}$$

### High-pass filter, derivative element

Allow frequencies to pass virtually unaffected above their cut-off frequency $f_C$.
Lower frequencies are dampened.




$$\frac{U_{out}}{U_{in}} = \frac{1}{\sqrt{1 + (f_C/f)^2}}$$

$$U_{out} = U_{in} \frac{R}{\sqrt{R^2 + X_C^2}}$$

$$U_{out} = U_{in} \frac{X_L}{\sqrt{R^2 + X_L^2}}$$

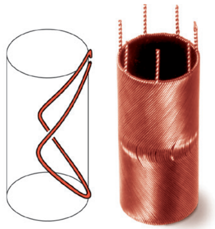
| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $C$ | Capacitance | F | $U_{out}$ | Output voltage | V |
| $I$ | Current | A | $U_R$ | Voltage over resistance | V |
| $L$ | Inductance | H | $X_C$ | Reactance, capacitive | Ω |
| $R$ | Electrical resistance | Ω | $X_L$ | Reactance, inductive | Ω |
| $U_C$ | Voltage over capacitance | V | $f$ | Frequency | Hz |
| $U_{in}$ | Input voltage | V | $f_C$ | Cut-off frequency | Hz |
| $U_L$ | Voltage over inductance | V | $\varphi$ | Phase shift | ° |

# 6. maxon motors
## 6.1 General

### What is special about maxon motors?

The **heart** of the maxon motor is the **self-supporting ironless copper winding**.

Outstanding features of the maxon DC motors:
- High efficiency → Low power consumption
- Very low moment of inertia → Highest acceleration
- Low inductance → Long service life
- Compact design → Good volume / power ratio
- No magnetic cogging
- Low electromagnetic interference
- High reliability



### maxon DC motor (brushed permanent-magnet energized DC motors)

| | | |
|---|---|---|
| **DCX range**<br>– Online configurable<br>– High performance thanks to NdFeB-Magnet<br>– Dynamic and high efficiency |  | *∅6 – 35 mm* |
| **DC-max range**<br>– Online configurable<br>– High performance at low costs<br>– Combines design of the A-max motors with NdFeB magnets<br>– Automated manufacturing process |  | *∅16 – 26 mm* |
| **RE range**<br>– High power density<br>– High-quality DC motor with NdFeB magnet<br>– High speeds and torques<br>– Robust design (metal flange) |  | *∅6 -65 mm* |
| **A-max range**<br>– Good price / performance ratio<br>– DC motor with AlNiCo magnet<br>– Automated manufacturing process |  | *∅12 – 32 mm* |

### Properties of the two brush systems

**Graphite brushes**

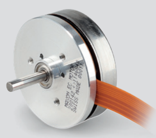
- Well suited for high currents and peak currents
- Well suited for start-stop and reverse operation
- Larger motors (from approx. 10 W)

- Higher friction, higher no-load current
- Not suited for low currents
- Higher audible noise
- Higher electromagnetic emissions
- More complex and higher costs

**Precious metal brushes**

- Well suited for lowest currents and voltages
- Well suited for continuous operation
- Smaller motors
- Very low friction, low audible noise
- Low electromagnetic emissions
- Cost effective

- Not suited for high currents and peak currents
- Not suited for start-stop operation

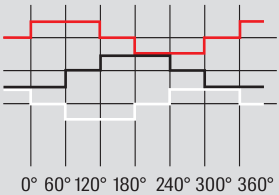
| maxon EC motor | | |
|---|---|---|
| Brushless DC motors (BLDC motors)<br>– Motor behavior similar to brushed DC motor<br>– Design similar to synchronous motor (3-phase stator winding, rotating permanent magnet)<br>– Powering of the 3 phases according to the rotor position by a commutation electronics | | |
| **ECX range**<br>– Online configurable<br>– Power-optimized, with high speeds up to 120 000 rpm<br>– Robust design<br>– Various types: e.g. short–long, sterilizable |  | *∅6 – 22 mm* |
| **EC range**<br>– Power-optimized, with high speeds up to 100 000 rpm<br>– Robust design<br>– Various types: e.g. short–long, sterilizable<br>– Lowest residual imbalance |  | *∅4 – 60 mm* |
| **EC-max range**<br>– Attractive price / performance ratio<br>– Robust steel housing<br>– Speeds up to 20 000 rpm<br>– Rotor with one pole pair |  | *∅16 – 40 mm* |
| **EC-4pole range**<br>– Highest power density thanks to 4-pole rotor<br>– Speeds up to 25 000 rpm<br>– Mechanical time constants below 3 ms. |  | *∅22 – 32 mm* |
| **EC flat motor range**<br>– Attractive price / performance ratio<br>– High torques due to external, multipole rotor<br>– Excellent heat dissipation at higher speeds, resulting from the to open design<br>– Speeds of up to 25 000 rpm |  | *∅9.2–90 mm* |
| **EC-i range**<br>– Highly dynamic due to internal, multipole rotor<br>– Mechanical time constants below 3 ms<br>– High torque density<br>– Speeds of up to 15 000 rpm |  | *∅30 – 52 mm* |
| **EC frameless range**<br>– High torque grace to multi-pole motor design<br>– Installation instructions with detailed specification for optimum integration.<br>– Sensor for supervising the temperature (NTC hot conductor)<br>– Space saving integration |  | *∅45 – 90 mm* |

## Electronic commutation

| Commutation type | Rotor position determination | |
|---|---|---|
| **Block commutation** | with Hall sensors | sensorless |

### Block commutation

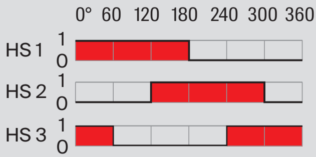
Block-shaped phase currents



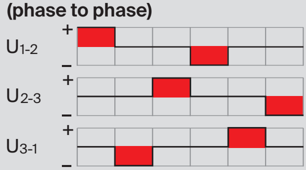

Turning angle

### with Hall sensors

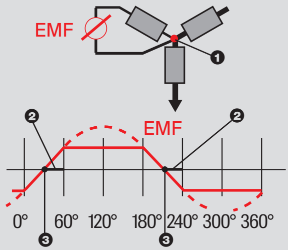
**Signal sequence diagram for the Hall sensors (HS)**




**Supplied motor voltage (phase to phase)**




### sensorless

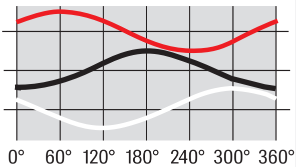



**Legend**
❶ Star point
❷ Time delay 30°
❸ Zero crossing of EMF

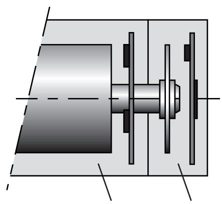
### Sinusoidal commutation

With encoder and Hall sensors (HS)

Sinusoidal phase currents




Turning angle



EC motor with HS    Encoder

## Comparison of DC and EC motors

**DC motor (brushed)**
- Simple operation and control, even without electronics
- No electronic parts in the motor

- Operating life limited by brush system
- Max. speeds limited by brush system

**EC motor (brushless)**
- Long operating life and high speeds with preloaded ball bearings
- No commutator arcing

- Iron losses in the magnetic return
- Needs electronics for operation (more cables and higher costs)
- Electronic parts in the motor (Hall sensors)

## 6.2 Power consideration DC motor

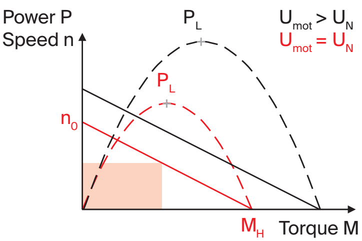
### Motor as energy converter


$$P_{el} = U_{mot} \cdot I_{mot}$$

$$P_L = \frac{\pi}{30} n \cdot M$$

$$P_J = R_{mot} \cdot I_{mot}^2$$

| | |
|---|---|
| Power balance, motor | $P_{el} = P_L + P_J$<br>$U_{mot} \cdot I_{mot} = \frac{\pi}{30} \cdot n \cdot M + R_{mot} \cdot I_{mot}^2$ |




In the speed-torque diagram, the output power is equivalent to the area of the rectangle below the speed-torque line. This rectangle is largest at half the stall torque and half the no-load speed.
The power curve is a parabola, whose maximum value is proportional to the square of the motor voltage.

| Symbol | Name | SI |
|---|---|---|
| $I_{mot}$ | Motor current | A |
| $M$ | Torque | Nm |
| $M_H$ | Stall torque | Nm |
| $P$ | Power | W |
| $P_{el}$ | Electrical input power | W |
| $P_J$ | Joule power loss | W |
| $P_L$ | Mechanical output power | W |

| Symbol | Name | SI |
|---|---|---|
| $R_{mot}$ | Terminal resistance, motor (catalog value) | Ω |
| $U_{mot}$ | Motor voltage | V |
| $U_N$ | Nominal voltage, motor (catalog value) | V |

| Symbol | Name | maxon |
|---|---|---|
| $n$ | Speed of rotation | rpm |
| $n_0$ | No load speed | rpm |

## 6.3 Motor constants and diagrams

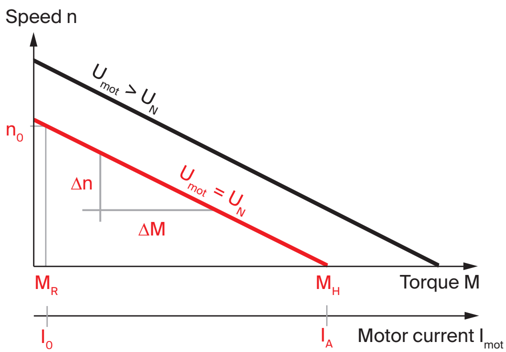
### Motor constants

The **speed constant $k_n$** and the **torque constant $k_M$** are two important characteristic values for the energy conversion.

| | |
|---|---|
| **Speed constant $k_n$**<br>The speed constant $k_n$ combines the speed $n$ with the voltage induced in the winding $U_{ind}$ (=EMF). | $n = k_n \cdot U_{ind}$ |
| **Torque constant $k_M$**<br>The torque constant $k_M$ links the produced torque $M$ with the electrical current $I$.<br><br>**Information:** maxon unit mNm/A | $M = k_M \cdot I_{mot}$ |
| **Dependence between $k_n$ and $k_M$** | $k_n \cdot k_M = \frac{30\,000}{\pi} \left[ \frac{\text{rpm}}{\text{V}} \cdot \frac{\text{mNm}}{\text{A}} \right] = 1 \left[ \frac{\text{rad}}{\text{s} \cdot \text{V}} \cdot \frac{\text{Nm}}{\text{A}} \right]$ |

### Speed-torque line

Describes the motor behavior — i.e. possible operating points ($n$, $M$) — at a constant voltage $U_{mot}$




$$n_0 \approx k_n \cdot U_{mot}$$

$$M_H = k_M \cdot I_A$$

$$M_R = k_M \cdot I_0$$

$$n = k_n \cdot U_{mot} - \frac{\Delta n}{\Delta M} \cdot M$$

(maxon units)

Speed / torque gradient:

$$\frac{\Delta n}{\Delta M} = \frac{30\,000}{\pi} \cdot \frac{R_{mot}}{k_M^{\,2}} \approx \frac{n_0}{M_H}$$

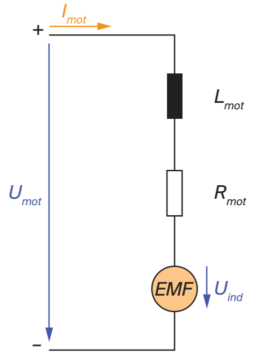
(maxon units)

| Symbol | Name | SI |
|---|---|---|
| $I_{mot}$ | Motor current | A |
| $I_A$ | Starting current | A |
| $I_0$ | No load current | A |
| $k_M$ | Torque constant (catalog value) | Nm/A |
| $M$ | Torque | Nm |
| $M_H$ | Stall torque | Nm |
| $M_R$ | Friction torque | Nm |
| $R_{mot}$ | Terminal resistance, motor (catalog value) | Ω |
| $U_{ind}$ | Induced voltage | V |

| Symbol | Name | SI |
|---|---|---|
| $U_{mot}$ | Motor voltage | V |
| $U_N$ | Nominal voltage, motor (catalog value) | V |

| Symbol | Name | maxon |
|---|---|---|
| $k_n$ | Speed constant (catalog value) | rpm/V |
| $n$ | Speed of rotation | rpm |
| $n_0$ | No load speed | rpm |
| $\Delta n / \Delta M$ | Speed / torque gradient, motor (catalog value) | rpm/mNm |

## Voltage equation, motor




$$U_{mot} = L_{mot} \cdot \frac{di}{dt} + R_{mot} \cdot i_{mot} + U_{ind} \cong R_{mot} \cdot I_{mot} + U_{ind}$$

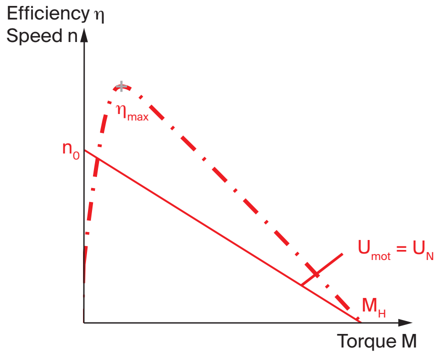
(at slow current change)

Derived from this the speed of rotation as a function of the load (speed-torque line)

$$n = k_n \cdot U_{mot} - \frac{\Delta n}{\Delta M} \cdot M = n_0 - \frac{\Delta n}{\Delta M} \cdot M$$

(maxon units)

## Efficiency curve in function of the torque




$$\eta = \frac{\pi}{30\,000} \cdot \frac{n \cdot (M - M_R)}{U_{mot} \cdot I_{mot}} \quad \text{(with } M_R = k_M \cdot I_0\text{)}$$

$$\eta_{max} = \left(1 - \sqrt{\frac{I_0}{I_A}}\right)^2$$

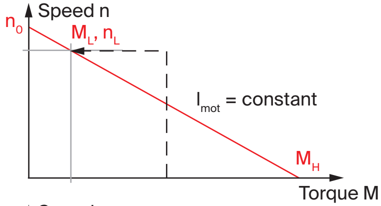
| Symbol | Name | SI |
|---|---|---|
| $EMF$ | Electromotive force | V |
| $I_0$ | No load current | A |
| $I_A$ | Starting current | A |
| $I_{mot}$, $i_{mot}$ | Motor current | A |
| $k_M$ | Torque constant (catalog value) | Nm/A |
| $L_{mot}$ | Terminal inductance, motor (catalog value) | H |
| $M$ | Torque | Nm |
| $M_H$ | Stall torque | Nm |
| $M_R$ | Friction torque | Nm |
| $R_{mot}$ | Terminal resistance, motor (catalog value) | Ω |
| $U_N$ | Nominal voltage, motor (catalog value) | V |
| $U_{ind}$ | Induced voltage | V |

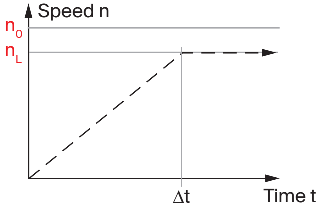
| Symbol | Name | SI |
|---|---|---|
| $U_{mot}$ | Motor voltage | V |
| $di$ | Current change | A |
| $dt$ | Time change | s |
| $\eta$ | Efficiency | |
| $\eta_{max}$ | Maximum efficiency at $U_N$ (catalog value) | |

| Symbol | Name | maxon |
|---|---|---|
| $k_n$ | Speed constant (catalog value) | rpm/V |
| $n$ | Speed of rotation | rpm |
| $n_0$ | No load speed | rpm |
| $\Delta n / \Delta M$ | Speed / torque gradient, motor (catalog value) | rpm/mNm |

## 6.4 Acceleration

### Angular acceleration: Start with constant current



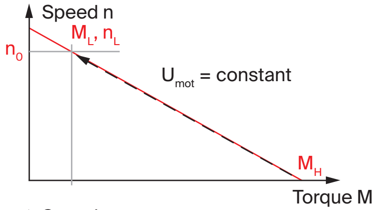

Acceleration

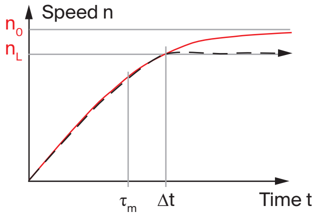
$$\alpha = \frac{M}{J_R + J_L} = \frac{k_M \cdot I_{mot}}{J_R + J_L}$$

Ramp time to load speed

$$\Delta t = \frac{\pi}{30} \cdot n_L \cdot \frac{J_R + J_L}{M} = \frac{\pi}{30} \cdot n_L \cdot \frac{J_R + J_L}{k_M \cdot I_{mot}}$$

### Angular acceleration: Start with constant terminal voltage




Acceleration, maximum

$$\alpha_{max} = \frac{M_H}{J_R + J_L}$$

Ramp time to load speed

$$\Delta t = \tau_m' \cdot \ln \left[ \frac{\left(1 - \frac{M_L + M_R}{M_H}\right) \cdot n_0}{\left(1 - \frac{M_L + M_R}{M_H}\right) \cdot n_0 - n_L} \right]$$

Mechanical time constant with load inertia

$$\tau_m' = \frac{(J_R + J_L) \cdot R_{mot}}{k_M^{\,2}}$$

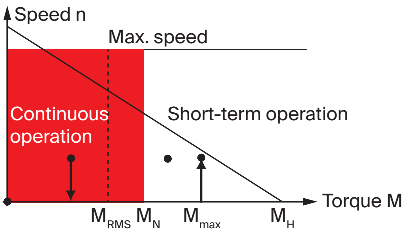
| Symbol | Name | SI |
|---|---|---|
| $I_{mot}$ | Motor current | A |
| $J_L$ | Moment of inertia, load | kgm² |
| $J_R$ | Moment of inertia, rotor (catalog value) | kgm² |
| $k_M$ | Torque constant (catalog value) | Nm/A |
| $M$ | Torque | Nm |
| $M_H$ | Stall torque | Nm |
| $M_L$ | Load torque | Nm |
| $M_R$ | Friction torque | Nm |
| $R_{mot}$ | Terminal resistance, motor (catalog value) | Ω |
| $t$ | Time | s |
| $U_{mot}$ | Motor voltage | V |

| Symbol | Name | SI |
|---|---|---|
| $\alpha$ | Angular acceleration | rad/s² |
| $\alpha_{max}$ | Maximum angular acceleration | rad/s² |
| $\Delta t$ | Acceleration time | s |
| $\tau_m$ | Mechanical time constant (catalog value) | s |
| $\tau_m'$ | Mechanical time constant with additional $J_L$ | s |

| Symbol | Name | maxon |
|---|---|---|
| $n$ | Speed of rotation | rpm |
| $n_0$ | No load speed | rpm |
| $n_L$ | Load speed | rpm |

## 6.5 Motor selection

### Motor type selection




Motor type selection based on the required torques

$M_N > M_{RMS}$

$M_H > M_{max}$

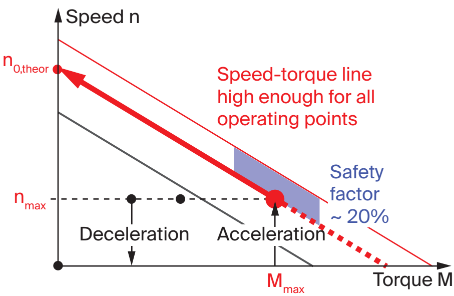
Root mean square load (RMS)

$$M_{RMS} = \sqrt{\frac{1}{t_{tot}}(t_1 \cdot M_1^2 + t_2 \cdot M_2^2 + \ldots + t_n \cdot M_n^2)}$$

**Remark:**
A motor type (e.g. DCX 32 L) is defined by: its size, the mechanical output power, the bearing system of the shaft, the commutation system used and the possible combinations with gearheads and sensors (maxon modular system)

### Winding selection

For an optimum match between the electrical and mechanical power components of the motor.




$k_n$ specifies the winding:
Select winding with

$$k_n > k_{n,theor} = \frac{n_{0,theor}}{U_{mot}} = \frac{n_{max} + \frac{\Delta n}{\Delta M} \cdot M_{max}}{U_{mot}}$$

(maxon units)

where $n_{max}$, $M_{max}$ is the extreme operating point and $\Delta n / \Delta M$ the average speed / torque gradient of the selected motor type.

**Recommendation:** Add a safety factor of approx. 20% to $k_n$ to compensate for tolerances and load changes; but do not select too large a value for $k_n$, as this would lead to large currents.

Required maximum motor current

$$I_{mot} = I_0 + \frac{M_{max}}{k_M}$$

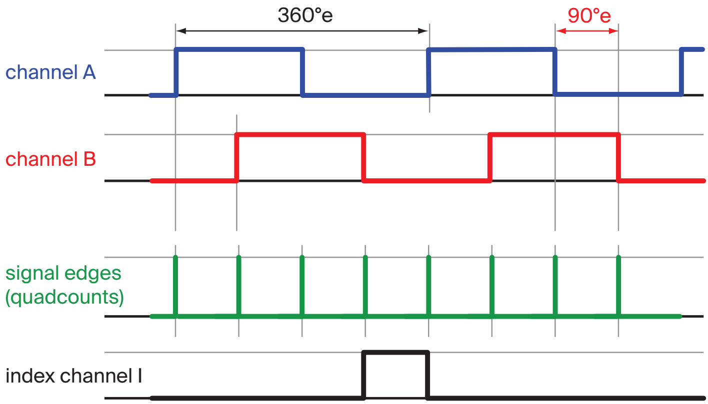
| Symbol | Name | SI |
|---|---|---|
| $I_{mot}$ | Motor current | A |
| $I_0$ | No load current | A |
| $k_M$ | Torque constant (catalog value) | Nm/A |
| $M_{1 \ldots n}$ | Torque at operating points 1…*n* | Nm |
| $M$ | Torque | Nm |
| $M_H$ | Stall torque | Nm |
| $M_N$ | Nominal torque, motor (catalog value) | Nm |
| $M_{RMS}$ | RMS torque | Nm |
| $M_{max}$ | Maximum torque in load cycle | Nm |
| $n$ | Speed of rotation | rpm |
| $U_{mot}$ | Motor voltage | V |

| Symbol | Name | SI |
|---|---|---|
| $t_{1 \ldots n}$ | Duration of operating points 1…*n* | s |
| $t_{tot}$ | Total time, operating cycle | s |

| Symbol | Name | maxon |
|---|---|---|
| $k_n$ | Speed constant (catalog value) | rpm/V |
| $k_{n,theor}$ | Required speed constant | rpm/V |
| $n$ | Speed of rotation | rpm |
| $n_{max}$ | Maximum speed in load cycle | rpm |
| $n_{0,theor}$ | Required no load speed | rpm |
| $\Delta n / \Delta M$ | Speed / torque gradient, motor (catalog value) | rpm/mNm |

# 7. maxon sensor

## maxon incremental encoder




## Recommended applications

| Principle | magnetic | | | | inductive | optical | | |
|---|---|---|---|---|---|---|---|---|
| Type | MR | EASY | QUAD | MEnc | MILE | HEDS, HEDL, AEDL | RIO | ENC22 |
| High number of counts | ✔ | ✔ | ✘ | ✘ | ✔ | ✔ | ✔ | ✓ |
| High speeds | ✔ | ✔ | ✔ | ✔ | ✔ | ✘ | ✔ | ✘ |
| Low speeds | ✓ | ✓ | ✘ | ✘ | ✔ | ✔ | ✔ | ✓ |
| Line driver (in the case of long cables, rough ambient conditions, positioning applications) | ✔ | ✔ | ✘ | ✘ | ✔ | ✔, ✘ | ✔ | ✘ |
| Low positioning accuracy or positioning with gearhead | ✔ | ✔ | ✘, ✓ | ✔ | ✔ | ✔ | ✔ | ✔ |
| High positioning accuracy | ✘, ✓ | ✓ | ✘ | ✘ | ✓ | ✔ | ✔ | ✓ |
| Index channel (for precision homing) | ✔ | ✔ | ✘ | ✘ | ✘ | ✔ | ✔ | ✘ |
| Dust, dirt, oil | ✔ | ✔ | ✔ | ✔ | ✔ | ✘ | ✔ | ✘ |
| Ionizing radiation | ✘ | ✘ | (✓) | (✓) | ✘ | ✘ | ✘ | ✘ |
| External magnetic fields | ✘ | (✓) | ✘ | ✘ | ✔ | ✔ | ✔ | ✔ |
| Mechanically robust | ✘ | ✔ | ✔ | ✘ | ✔ | ✘ | ✔ | ✘ |

✔ Recommended  ✓ With restrictions  (✓) Optional (on request)  ✘ Not recommended

## Counts per turn from position resolution

| | |
|---|---|
| Required counts per turn $N$ of the encoder for a specified position resolution of $\Delta\varphi$ on the output of a backlash free mechanical drive. | $N \geq \dfrac{360°}{\Delta\varphi \cdot i}$ |

**Remark:** By evaluating the quadcounts (*qc*), a four times finer resolution is achieved. This is recommended for a sufficiently accurate positioning.

## Measurement resolution, motor speed

| | |
|---|---|
| **Example:**<br>Measurement resolution $\Delta Q$: (given by the sample rate of the speed controller) 1 qc/ms<br>Counts per turn $N$, encoder: 500 CPT | $\Delta n = \dfrac{\Delta Q}{Q \cdot N}$<br>$\Delta n = \dfrac{\Delta Q}{Q \cdot N} = \dfrac{1\ \dfrac{\text{qc}}{\text{ms}}}{4\ \dfrac{\text{qc}}{\text{CPT}} \cdot 500\ \text{CPT}} = \dfrac{60\,000\ \dfrac{\text{qc}}{\text{min}}}{2000\ \text{qc}} = 30\ \text{rpm}$ |

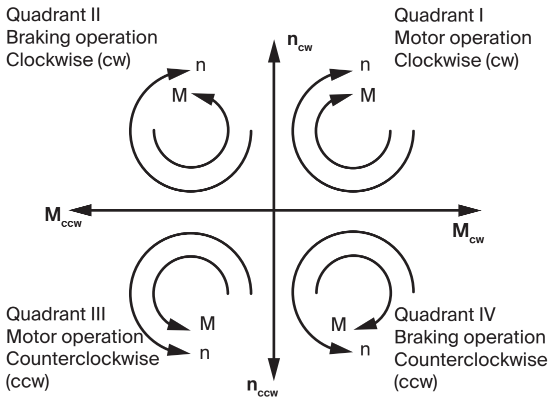
**Comment:** The achievable speed stability is much higher than the above measurement resolution, due to the mass inertias and feed forward (if applicable).

| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $N$ | Counts per turn | CPT | $\Delta\varphi$ | Position resolution | ° |
| $i$ | Reduction ratio, mechanical drive | | | | |
| $Q = 4$ | Quadcounts per pulse | qc/IMP | **Symbol** | **Name** | **maxon** |
| $\Delta Q$ | Measurement resolution | qc/ms | $\Delta n$ | Measurement resolution, motor speed | rpm |

# 8. maxon controller

## 8.1 Operating quadrants

**Operating quadrants**




**1-Q operation**
- Only motor operation (Quadrant I or Quadrant III)
- Direction reversal via digital signal
- Typical: amplifier for EC motors
- Braking is not controlled (friction), often slow

**4-Q operation**
- Controlled motor operation and braking operation in both rotation directions (all 4 quadrants)
- A must for positioning tasks

## 8.2 Selection of power supply

**Required supply voltage at given load *($n_L$, $M_L$)***

$$V_{CC} \geq \frac{U_N}{n_{0,UN}} \cdot \left(n_L + \frac{\Delta n}{\Delta M} \cdot M_L\right) + \Delta U_{max} \quad \text{(maxon units)}$$

**Notes:**
- In the case of a 4Q servo amplifier, the power supply has to be able to absorb the kinetic energy generated (for example in a capacitor) when the load is decelerated.
- When a stabilized power supply is used, the overcurrent protection has to be deactivated for the operating range.
- The formula includes the maximum voltage drop $\Delta U_{max}$ of the controller at maximum continuous current.

**Achievable speed at given voltage supply**

$$n_L \leq \left[(V_{CC} - \Delta U_{max}) \cdot \frac{n_{0,UN}}{U_N}\right] - \left[\frac{\Delta n}{\Delta M} \cdot M_L\right] \text{(maxon units)}$$

| Symbol | Name | SI |
|---|---|---|
| $M$ | Torque | Nm |
| $M_L$ | Load torque | Nm |
| $U_N$ | Nominal voltage, motor (catalog value) | V |
| $V_{CC}$ | Supply voltage | V |
| $\Delta U_{max}$ | Maximum voltage drop of the controller | V |

| Symbol | Name | maxon |
|---|---|---|
| $n$ | Speed of rotation | rpm |
| $n_L$ | Load speed | rpm |
| $n_{0,UN}$ | No load speed motor at $U_N$ (catalog value) | rpm |
| $\Delta n / \Delta M$ | Speed / torque gradient, motor (catalog value) | rpm/mNm |

## 8.3 Size of the motor choke with PWM controllers

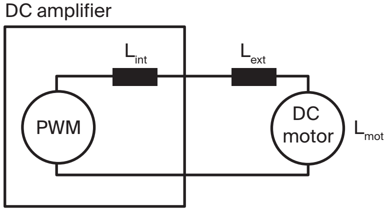
| **Calculation of current ripple** | | | |
|---|---|---|---|
| **PWM scheme** | **1-Q** | **2-level (4-Q)** | **3-level (4-Q)** |
| Maximum current ripple, peak-to-peak | $\Delta I_{PP,max} = \frac{V_{CC}}{4 \cdot L_{tot} \cdot f_{PWM}}$ | $\Delta I_{PP,max} = \frac{V_{CC}}{2 \cdot L_{tot} \cdot f_{PWM}}$ | $\Delta I_{PP,max} = \frac{V_{CC}}{4 \cdot L_{tot} \cdot f_{PWM}}$ |
| Calculation $L_{tot}$ | $L_{tot} = L_{int} + 0.3...0.8 \cdot L_{mot} + L_{ext}$ | | |

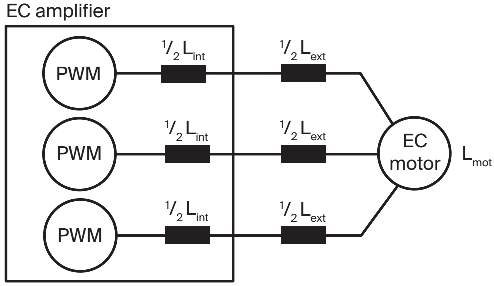
The effective motor inductance in the case of square PWM excitation only amounts to approx. 30–80% of the catalog value $L_{mot}$.
The catalog value $L_{mot}$ is defined at a frequency of 1 kHz with sinusoidal excitation.

- At a current ripple of $\Delta I_{PP} \leq 1.5 \cdot I_N$ the motor can still be loaded to approx. 90% of the nominal current $I_N$ (catalog value).
- At a current ripple of $\Delta I_{PP} > 1.5 \cdot I_N$, it is recommended to use an external motor choke, in accordance with the formula below.

| **Calculation, additional external motor choke** | | |
|---|---|---|
| **PWM scheme** | **1-Q and 3-level (4-Q)** | **2-level (4-Q)** |
| Rule of thumb | $L_{ext} = \frac{V_{CC}}{6 \cdot I_N \cdot f_{PWM}} - L_{int} - 0.3 \cdot L_{mot}$ | $L_{ext} = \frac{V_{CC}}{3 \cdot I_N \cdot f_{PWM}} - L_{int} - 0.3 \cdot L_{mot}$ |

$L_{ext} \leq 0$ No additional motor choke required
$L_{ext} > 0$ Additional motor choke recommended

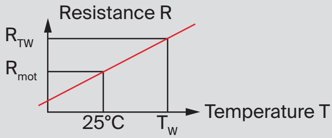
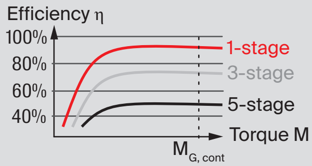



| Symbol | Name | SI | Symbol | Name | SI |
|---|---|---|---|---|---|
| $f_{PWM}$ | PWM frequency | Hz | $L_{mot}$ | Terminal inductance, motor (catalog value) | H |
| $I_N$ | Nominal current, motor (catalog value) | A | $L_{tot}$ | Total inductance | H |
| $L_{ext}$ | Inductance, additional external motor choke | H | $V_{CC}$ | Supply voltage | V |
| | | | $\Delta I_{PP}$ | Current ripple, peak-to-peak | A |
| $L_{int}$ | Inductance, built-in choke controller | H | $\Delta I_{PP,max}$ | Maximum current ripple, peak-to-peak | A |

# 9. Thermal behavior
## 9.1 Basics

### Heat sources

| | | |
|---|---|---|
| **Iron losses in EC motors and motors with iron core winding** | Remagnetization losses $P_{V,\,magn} = \frac{\pi}{30} \cdot n \cdot M_{magn}$ | Eddy current losses $P_{V,eddy} = const \cdot n^2$ |
| **Joule power losses in winding** |   | $P_J = R_{TW} \cdot I_{mot}^{\ 2}$ <br> $R_{TW} = R_{mot} \cdot [1 + \alpha_{Cu} \cdot (T_W - 25°C)]$ |
| **Friction losses: in the bearings and in the brushes (brushed DC motors)** | | |
| **Losses in the gearhead** |   | $P_{V,R} = \frac{\pi}{30} \cdot n_{mot} \cdot M_{mot} \cdot (1 - \eta_G)$ <br> $P_{V,R} = \frac{\pi}{30} \cdot n_L \cdot M_L \cdot \frac{1 - \eta_G}{\eta_G}$ |

### Stall torque reduced through temperature rise

| | |
|---|---|
| First approximation; calculated from voltage and increased winding resistance (Temperature dependence of $k_M$ not considered) | $M_{HT} = k_M \cdot I_{AT} = k_M \cdot \frac{U_{mot}}{R_{TW}}$ |

### Storing heat

| | | |
|---|---|---|
| $Q = c \cdot m \cdot \Delta T = C_{th} \cdot \Delta T$ | $Q = P_V \cdot t$ | |
| Winding: $C_{th,W} = c_{Cu} \cdot m_W$ | Stator: $C_{th,S} = c_{Fe} \cdot m_{mot}$ | Gearhead: $C_{th,G} = c_{Fe} \cdot m_G$ |

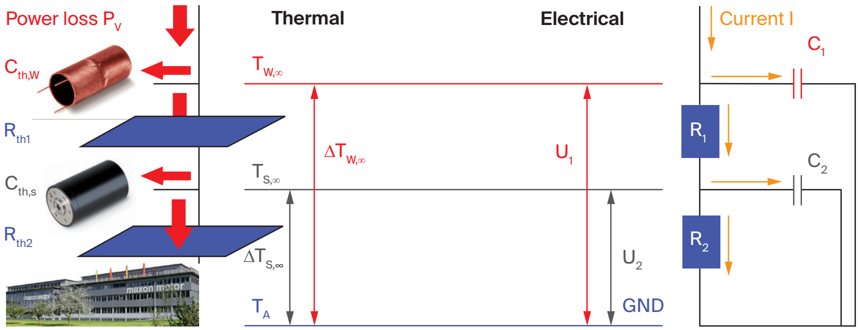
| Symbol | Name | SI |
|---|---|---|
| $C_{th}$ | Heat capacity | J/K |
| $C_{th,G}$ | Heat capacity gearhead | J/K |
| $C_{th,S}$ | Heat capacity stator | J/K |
| $C_{th,W}$ | Heat capacity winding | J/K |
| $c$ | Specific heat capacity | J/(kgK) |
| $I_{AT}$ | Starting current at temperature $T_W$ | A |
| $I_{mot}$ | Motor current | A |
| $k_M$ | Torque constant (catalog value) | Nm/A |
| $M$ | Torque | Nm |
| $M_{G,cont}$ | Maximum continuous torque, gearhead (catalog value) | Nm |
| $M_{HT}$ | Stall torque at temperature $T_W$ | Nm |
| $M_L$ | Load torque | Nm |
| $M_{magn}$ | Torque for reversal of magnetization | Nm |
| $M_{mot}$ | Motor torque | Nm |
| $m$ | Mass | kg |
| $m_G$ | Mass, gearhead | kg |
| $m_{mot}$ | Mass, motor | kg |
| $m_W$ | Mass, winding | kg |
| $P_J$ | Joule power losses | W |
| $P_V$ | Power losses | W |
| $P_{V,eddy}$ | Eddy current power losses | W |

| Symbol | Name | SI |
|---|---|---|
| $P_{V,magn}$ | Power losses for reversal of magnetization | W |
| $P_{V,R}$ | Friction power losses | W |
| $Q$ | Stored heat | J |
| $R_{mot}$ | Terminal resistance, motor (catalog value) | Ω |
| $R_{TW}$ | Winding resistance at current temp. $T_W$ | Ω |
| $T$ | Temperature | °C |
| $T_W$ | Winding temperature | °C |
| $t$ | Time | s |
| $U_{mot}$ | Motor voltage | V |
| $\Delta T$ | Temperature difference | K |
| $\eta$ | Efficiency | |
| $\eta_G$ | Gearhead efficiency | |

| Symbol | Name | maxon |
|---|---|---|
| $n$ | Speed of rotation | rpm |
| $n_L$ | Load speed | rpm |
| $n_{mot}$ | Motor speed | rpm |

| Symbol | Name | Value |
|---|---|---|
| $\alpha_{Cu}$ | Resistance coefficient, copper | 0.0039 $K^{-1}$ |
| $c_{Cu}$ | Specific heat capacity copper | 380 J/(kgK) |
| $c_{Fe}$ | Specific heat capacity iron | 450 – 470 J/(kgK) |

## Analogy to electrical circuit:



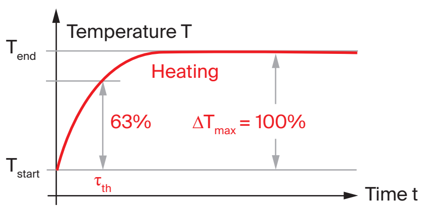

**Thermal** → *Heat flow*
Losses

| Symbol | Name | Unit |
|---|---|---|
| $Q$ | Stored heat | J |
| $P_V$ | Power losses | W = J/s |
| $\Delta T_{W,\infty}$ | Temperature difference, winding–ambient | K |
| $\Delta T_{S,\infty}$ | Temperature difference, stator–ambient | K |
| $T_A$ | Ambient temperature | °C (K) |
| $R_{th1}$ | Therm. resistance, winding–housing (catalog value) | K/W |
| $R_{th2}$ | Therm. resistance, housing–ambient (catalog value) | K/W |
| $C_{th,W}$ | Heat capacity, winding | J/K |
| $C_{th,S}$ | Heat capacity, stator | J/K |

**Electrical** → *Current flow*
Current source

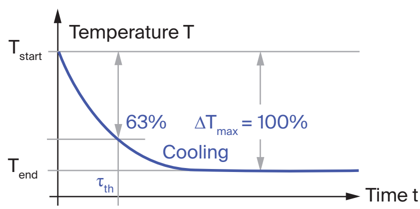
| Symbol | Name | Unit |
|---|---|---|
| $Q$ | Electric charge | C |
| $I$ | Current | A = C/s |
| $U_1$ | Voltage, potential difference | V |
| $U_2$ | Voltage, potential difference | V |
| $GND$ | Ground | V |
| $R_1$ | Electrical resistance | Ω |
| $R_2$ | Electrical resistance | Ω |
| $C_1$ | Electrical capacitance | F |
| $C_2$ | Electrical capacitance | F |

## Heating of a simple body




$$\Delta T(t) = \Delta T_{max} \cdot \left[1 - e^{-\frac{t}{\tau_{th}}}\right]$$

## Cooling of a simple body




$$\Delta T(t) = \Delta T_{max} \cdot e^{-\frac{t}{\tau_{th}}}$$

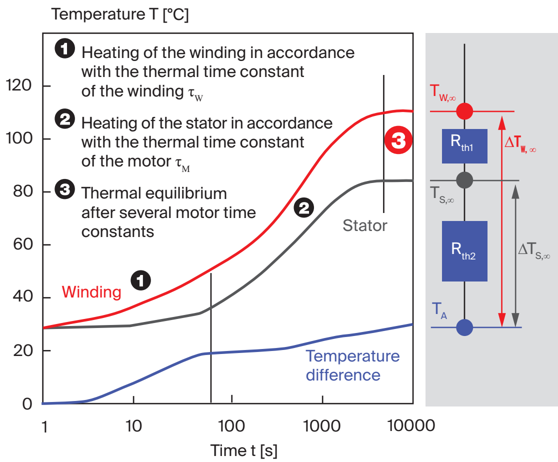
| Symbol | Name | SI |
|---|---|---|
| $T$ | Temperature | °C |
| $T_A$ | Ambient Temperature | °C |
| $T_{end}$ | End temperature | °C |
| $T_{start}$ | Temperature at start | °C |
| $T_{S,\infty}$ | End temperature, stator | °C |
| $T_{W,\infty}$ | End temperature, winding | °C |

| Symbol | Name | SI |
|---|---|---|
| $t$ | Time | s |
| $\Delta T_{max}$ | Maximum temperature change | K |
| $\Delta T(t)$ | Temperature change as a function of time $t$ | K |
| $\tau_{th}$ | Thermal time constant | s |

## 9.2 Continuous operation

Continuous operation is characterized by a thermal equilibrium. After several motor time constants the temperature difference between the rotor and stator stays constant, as their temperatures do not increase further.

### Motor: Winding and stator temperature

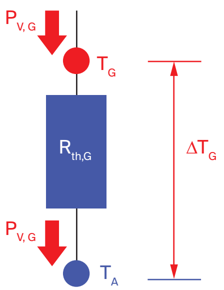



$$\Delta T_{W,\infty} = T_{W,\infty} - T_A = (R_{th1} + R_{th2}) \cdot P_J$$

$$\Delta T_{W,\infty} = \frac{(R_{th1} + R_{th2}) \cdot R_{TA} \cdot I_{mot}^{\ 2}}{1 - \alpha_{Cu} \cdot (R_{th1} + R_{th2}) \cdot R_{TA} \cdot I_{mot}^{\ 2}}$$

$$\Delta T_{S,\infty} = T_{S,\infty} - T_A = \frac{R_{th2}}{R_{th1} + R_{th2}} \Delta T_{W,\infty}$$

### Gearhead: Housing temperature




$$\Delta T_G = T_G - T_A = R_{th,G} \cdot P_{V,G}$$

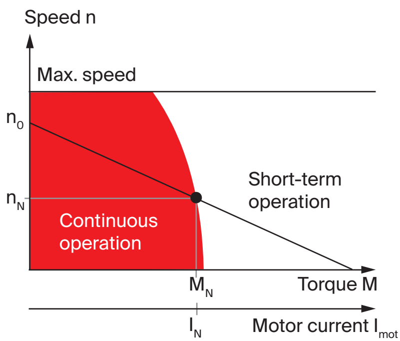
$R_{th,\,G}$: e.g. estimated with $R_{th2}$ of motors with the same size

| Symbol | Name | SI |
|---|---|---|
| $I_{mot}$ | Motor current | A |
| $P_J$ | Joule power losses | W |
| $P_{V,G}$ | Power losses, gearhead | W |
| $R_{TA}$ | Winding resistance at temperature $T_A$ | Ω |
| $R_{th,G}$ | Therm. resistance, gearhead–ambient | K/W |
| $R_{th1}$ | Therm. resistance, winding–housing (catalog value) | K/W |
| $R_{th2}$ | Therm. resistance, housing–ambient (catalog value) | K/W |
| $T$ | Temperature | °C |
| $T_A$ | Ambient temperature | °C |
| $T_G$ | Gearhead temperature | °C |

| Symbol | Name | SI |
|---|---|---|
| $T_{S,\infty}$ | End temperature, stator | °C |
| $T_{W,\infty}$ | End temperature, winding | °C |
| $t$ | Time | s |
| $\Delta T_G$ | Temperature difference, gearhead–ambient | K |
| $\Delta T_{S,\infty}$ | Temperature difference, stator–ambient | K |
| $\Delta T_{W,\infty}$ | Temperature difference, winding–ambient | K |
| $\tau_M$ | Therm. time constant, motor (catalog value) | s |
| $\tau_W$ | Therm. time constant, winding (catalog value) | s |

| Symbol | Name | Value |
|---|---|---|
| $\alpha_{Cu}$ | Resistance coefficient, copper | 0.0039 $K^{-1}$ |

## Permissible nominal current $I_N$



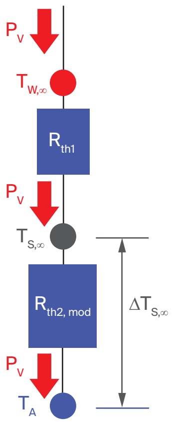

Dependency on ambient temperature under standard mounting conditions (free air convection at 25°C; mounted horizontally on plastic plate)

$$I_{N,TA} = I_N \cdot \sqrt{\frac{T_{max} - T_A}{T_{max} - 25°C}}$$

Temperature-dependence under modified mounting conditions

$$I_{N,TA} = I_N \cdot \sqrt{\frac{T_{max} - T_A}{T_{max} - 25°C} \cdot \frac{R_{th1} + R_{th2}}{R_{th1} + R_{th2,mod}}}$$

## Determining $R_{th2,mod}$




**Motor under original conditions**
- Installation, fastening, air circulation

**Separate measurement in continuous operation**
at any motor current $I_{mot}$
- Stator temperature $T_{S,}$
- Ambient temperature $T_A$

$$R_{th2,mod} = \Delta T_{S,\infty} \cdot \frac{1 - \alpha_{Cu} \cdot R_{th1} \cdot R_{TA} \cdot I_{mot}^2}{R_{TA} \cdot I_{mot}^2 \cdot (1 + \alpha_{Cu} \cdot \Delta T_{S,\infty})}$$

| Symbol | Name | SI |
|---|---|---|
| $I_{mot}$ | Motor current | A |
| $I_N$ | Nominal current, motor (catalog value) | A |
| $I_{N,TA}$ | Nominal current as a function of $T_A$ | A |
| $M$ | Torque | Nm |
| $M_N$ | Nominal torque, motor (catalog value) | Nm |
| $P_V$ | Power losses | W |
| $R_{TA}$ | Winding resistance at temperature $T_A$ | Ω |
| $R_{th1}$ | Therm. resistance, winding–housing (catalog value) | K/W |
| $R_{th2}$ | Therm. resistance, housing–ambient (catalog value) | K/W |
| $R_{th2,mod}$ | Therm. resistance, housing–ambient modified | K/W |
| $T_A$ | Ambient temperature | °C |

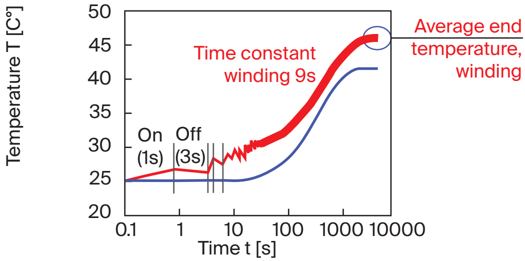
| Symbol | Name | SI |
|---|---|---|
| $T_{max}$ | Max. winding temperature (catalog value) | °C |
| $T_{S,\infty}$ | End temperature, stator | °C |
| $T_{W,\infty}$ | End temperature, winding | °C |
| $\Delta T_{S,\infty}$ | Temperature difference, stator–ambient | K |

| Symbol | Name | maxon |
|---|---|---|
| $n$ | Speed of rotation | rpm |
| $n_N$ | Nominal speed, motor (catalog value) | rpm |
| $n_O$ | No load speed | rpm |

| Symbol | Name | Value |
|---|---|---|
| $\alpha_{Cu}$ | Resistance coefficient, copper | 0.0039 $K^{-1}$ |

## 9.3 Cyclic and intermittent operation (continuously repeated)

Repetitive work cycles of short duration (typically only a few seconds) can be assessed with the same formalism as continuous operation.



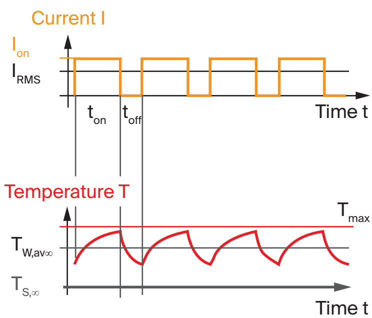

### Average temperature rise during intermittent operation

Use effective current value (RMS) as motor load.

$$\Delta T_{W,\infty} = \frac{(R_{th1} + R_{th2}) \cdot R_{TA} \cdot I_{RMS}^{2}}{1 - \alpha_{Cu}\,(R_{th1} + R_{th2}) \cdot R_{TA} \cdot I_{RMS}^{2}}$$

$$\Delta T_{S,\infty} = \frac{R_{th2}}{(R_{th1} + R_{th2})}\,\Delta T_{W,\infty}$$

### Intermittent operation




RMS current

$$I_{RMS} = I_{on} \cdot \sqrt{\frac{t_{on}}{t_{on} + t_{off}}}$$

Basic requirement: $I_{RMS} \leq I_{N,TA}$

Maximum load current for a given time cycle

$$I_{on} \leq I_N \cdot \sqrt{\frac{T_{max} - T_A}{T_{max} - 25°C} \cdot \frac{t_{on} + t_{off}}{t_{on}}}$$

OFF duration for a load of $I_{on}$ during $t_{on}$

$$t_{off} \geq \left[ \frac{I_{on}^{2}}{I_N^{2} \cdot \frac{T_{max} - T_A}{T_{max} - 25°C}} - 1 \right] \cdot t_{on}$$

| Symbol | Name | SI |
|---|---|---|
| $I$ | Current | A |
| $I_N$ | Nominal current, motor (catalog value) | A |
| $I_{N,TA}$ | Nominal current as a function of $T_A$ | A |
| $I_{on}$ | Current during ON phase | A |
| $I_{RMS}$ | *RMS* current | A |
| $R_{TA}$ | Winding resistance at temperature $T_A$ | Ω |
| $R_{th1}$ | Therm. resistance, winding–housing (catalog value) | K/W |
| $R_{th2}$ | Therm. resistance, housing–ambient (catalog value) | K/W |
| $T$ | Temperature | °C |
| $T_A$ | Ambient temperature | °C |

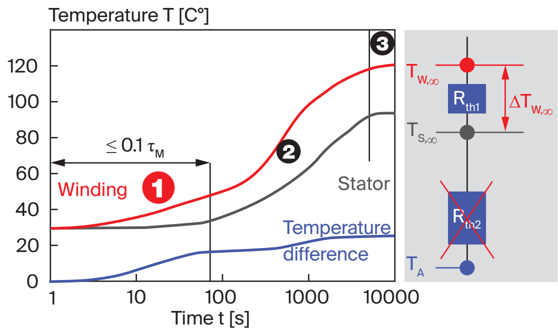
| Symbol | Name | SI |
|---|---|---|
| $T_{max}$ | Max. winding temperature (catalog value) | °C |
| $T_{S,\infty}$ | End temperature, stator | °C |
| $T_{W,av\infty}$ | Average end temperature, winding | °C |
| $t$ | Time | s |
| $t_{off}$ | OFF time | s |
| $t_{on}$ | ON time | s |
| $\Delta T_{S,\infty}$ | Temperature difference, stator–ambient | K |
| $\Delta T_{W,\infty}$ | Temperature difference, winding–ambient | K |

| Symbol | Name | Value |
|---|---|---|
| $\alpha_{Cu}$ | Resistance coefficient, copper | 0.0039 $K^{-1}$ |

## 9.4 Short-term operation

High, brief, one-time overload of the motor. The operation duration is so short that the temperature of the thermally inert stator does not increase significantly; this corresponds to an ON time of approx. $\tau_M/10$ ($\approx 5 \cdot \tau_W$).

→ Only the heating of the winding, which corresponds to the heating of a simple body (see chapter 9.1), has to be taken into account.




$$T_{S,\infty} = T_A$$

$$\Delta T_{W,\infty} = R_{th1} \cdot P_J$$

$$\Delta T_{W,\infty} = \frac{R_{th1} \cdot R_{TA} \cdot I_{mot}^2}{1 - \alpha_{Cu} \cdot R_{th1} \cdot R_{TA} \cdot I_{mot}^2}$$

### Overload factor K

Quantification of the overload

**Meaning:**

- K < 1: $T_{max}$ is not reached during short-term operation
- K > 1: Limit maximum ON time $t_{on}$

$$K = \frac{I_{mot}}{I_N} \cdot \sqrt{\frac{T_{max} - 25°C}{T_{max} - T_{S,\infty}} \cdot \frac{R_{th1}}{R_{th1} + R_{th2}}}$$

Maximum permissible overload at given ON time $t_{on}$

$$K = \sqrt{\frac{1}{1 - exp\left[-\frac{t_{on}}{\tau_W}\right]}}$$

Maximum ON time $t_{on}$ at given overload factor $K$

$$t_{on} = \tau_W \cdot ln \frac{K^2}{K^2 - 1}$$

Maximum motor current $I_{mot}$ at given overload factor $K$

$$I_{mot} = K \cdot I_N \cdot \sqrt{\frac{T_{max} - T_S}{T_{max} - 25°C} \cdot \frac{R_{th1} + R_{th2}}{R_{th1}}}$$

| Symbol | Name | SI |
|---|---|---|
| $I_{mot}$ | Motor current | A |
| $I_N$ | Nominal current, motor (catalog value) | A |
| $K$ | Overload factor | |
| $P_J$ | Joule power losses | W |
| $R_{TA}$ | Winding resistance at temperature $T_A$ | Ω |
| $R_{th1}$ | Therm. resistance, winding–housing (catalog value) | K/W |
| $R_{th2}$ | Therm. resistance, housing–ambient (catalog value) | K/W |
| $T$ | Temperature | °C |
| $T_A$ | Ambient temperature | °C |
| $T_{max}$ | Max. winding temperature (catalog value) | °C |

| Symbol | Name | SI |
|---|---|---|
| $T_{S,\infty}$ | End temperature, stator | °C |
| $T_{W,\infty}$ | End temperature, winding | °C |
| $t$ | Time | s |
| $t_{on}$ | ON time | s |
| $\Delta T_{W,\infty}$ | Temperature difference, winding–ambient | K |
| $\tau_M$ | Therm. time constant, motor (catalog value) | s |
| $\tau_W$ | Therm. time constant, winding (catalog value) | s |

| Symbol | Name | Value |
|---|---|---|
| $\alpha_{Cu}$ | Resistance coefficient, copper | 0.0039 K$^{-1}$ |

# 10. Tables
## 10.1 maxon Conversion Tables

### General Information

Quantities and their basic units in the International System of Units (SI)

| Quantity | Base unit | Unit sign |
|---|---|---|
| Length | Meter | m |
| Mass | Kilogram | kg |
| Time | Second | s |
| Electric current | Ampere | A |
| Thermodynamic temperature | Kelvin | K |

**Conversion example**
A Known unit
B Unit sought

| Known: | Multiply by | ought: |
|---|---|---|
| oz-in | 7.06 | mNm |

**Factors used for ...**

**... conversions:**
1 oz = $2.834952313 \cdot 10^{-2}$ kg
1 in = $2.54 \cdot 10^{-2}$ m

**... gravitational acceleration:**
g = 9.80665 m $s^{-2}$
= 386.08858 in $s^{-2}$

**... derived units:**
1 yd = 3 ft = 36 in
1 lb = 16 oz = 7000 gr (grains)
1 kp = 1 kg · 9.80665 $ms^{-2}$
1 N = 1 $kgms^{-2}$
1 W = 1 $Nms^{-1}$ = 1 $kgm^2s^{-3}$
1 J = 1 $Nms^{-1}$ = 1 Ws

### Decimal multiples and fractions of units

| Prefix | Abbre-viation | Power of ten | Prefix | Abbre-viation | Power of ten |
|---|---|---|---|---|---|
| deca.. | da | $10^1$ | deci.. | d | $10^{-1}$ |
| hecto.. | h | $10^2$ | centi.. | c | $10^{-2}$ |
| kilo.. | k | $10^3$ | milli.. | m | $10^{-3}$ |
| mega.. | M | $10^6$ | micro.. | m | $10^{-6}$ |
| giga.. | G | $10^9$ | nano.. | n | $10^{-9}$ |
| tera.. | T | $10^{12}$ | pico.. | p | $10^{-12}$ |

**Power** *P* [W]

| B \ A | oz-in-$s^{-1}$ | oz-in-rpm | in-lbf-$s^{-1}$ | ft-lbf-$s^{-1}$ | N$ms^{-1}$=W | mW | kpm $s^{-1}$ | mNmrpm |
|---|---|---|---|---|---|---|---|---|
| W=Nm $s^{-1}$ | $7.06 \cdot 10^{-3}$ | $1.17 \cdot 10^{-4}$ | 0.113 | 1.356 | – | $1 \cdot 10^{-3}$ | 9.807 | $^1/_{60000}$ |
| mW | 7.06 | 0.117 | 112.9 | $1.356 \cdot 10^3$ | $1 \cdot 10^3$ | – | $9.807 \cdot 10^3$ | $^1/_{60}$ |
| oz-in-$s^{-1}$ | – | $^1/_{60}$ | 16 | 192 | 141.6 | 0.142 | $1.39 \cdot 10^3$ | $2.36 \cdot 10^{-3}$ |
| ft-lbf-$s^{-1}$ | $^1/_{192}$ | $^1/_{11520}$ | $^1/_{12}$ | – | 0.737 | $0.737 \cdot 10^{-3}$ | 7.233 | $1.23 \cdot 10^{-5}$ |
| kpm $s^{-1}$ | $7.20 \cdot 10^{-4}$ | $1.2 \cdot 10^{-5}$ | $1.15 \cdot 10^{-2}$ | 0.138 | 0.102 | $0.102 \cdot 10^{-3}$ | – | $1.70 \cdot 10^{-6}$ |

**Torque** *M* [Nm]

| B \ A | oz-in | ft-lbf | Nm = Ws | Ncm | mNm | kpm | pcm | |
|---|---|---|---|---|---|---|---|---|
| Nm = $W_s$ | $7.06 \cdot 10^{-3}$ | 1.356 | – | $1 \cdot 10^{-2}$ | $1 \cdot 10^{-3}$ | 9.807 | $9.807 \cdot 10^{-5}$ | |
| mNm | 7.06 | $1.356 \cdot 10^3$ | $1 \cdot 10^3$ | 10 | – | $9.807 \cdot 10^3$ | $9.807 \cdot 10^{-2}$ | |
| kpm | $7.20 \cdot 10^{-4}$ | 0.138 | 0.102 | $0.102 \cdot 10^{-2}$ | $0.102 \cdot 10^{-3}$ | – | $1 \cdot 10^{-5}$ | |
| oz-in | – | 192 | 141.6 | 1.416 | 0.142 | $1.39 \cdot 10^3$ | $1.39 \cdot 10^{-2}$ | |
| ft-lbf | $^1/_{192}$ | – | 0.737 | $0.737 \cdot 10^{-2}$ | $0.737 \cdot 10^{-3}$ | 7.233 | $7.233 \cdot 10^{-5}$ | |

**Moment of inertia** *J* [kg $m^2$]

| B \ A | oz-$in^2$ | oz-in-$s^2$ | lb-$in^2$ | lb-in-$s^2$ | $Nms^2$=$kgm^2$ | mNm $s^2$ | $gcm^2$ | kpm $s^2$ |
|---|---|---|---|---|---|---|---|---|
| g $cm^2$ | 182.9 | $7.06 \cdot 10^4$ | $2.93 \cdot 10^3$ | $1.13 \cdot 10^6$ | $1 \cdot 10^7$ | $1 \cdot 10^4$ | – | $9.807 \cdot 10^7$ |
| $kgm^2$=$Nms^2$ | $1.83 \cdot 10^{-5}$ | $7.06 \cdot 10^{-3}$ | $2.93 \cdot 10^{-4}$ | 0.113 | – | $1 \cdot 10^{-3}$ | $1 \cdot 10^{-7}$ | 9.807 |
| oz-$in^2$ | – | 386.08 | 16 | $6.18 \cdot 10^3$ | $5.46 \cdot 10^4$ | 54.6 | $5.46 \cdot 10^{-3}$ | $5.35 \cdot 10^5$ |
| lb-$in^2$ | $^1/_{16}$ | 24.130 | – | 386.08 | $3.41 \cdot 10^3$ | 3.41 | $3.41 \cdot 10^{-4}$ | $3.35 \cdot 10^4$ |

**Mass** *m* [kg]

| B \ A | oz | lb | gr (grain) | kg | g |
|---|---|---|---|---|---|
| kg | $28.35 \cdot 10^{-3}$ | 0.454 | $64.79 \cdot 10^{-6}$ | – | $1 \cdot 10^{-3}$ |
| g | 28.35 | $0.454 \cdot 10^3$ | $64.79 \cdot 10^{-3}$ | $1 \cdot 10^3$ | – |
| oz | – | 16 | $2.28 \cdot 10^{-3}$ | 35.27 | $35.27 \cdot 10^3$ |
| lb | $^1/_{16}$ | – | $^1/_{7000}$ | 2.205 | $2.205 \cdot 10^3$ |
| gr (grain) | 437.5 | 7000 | – | $15.43 \cdot 10^3$ | $15.43 \cdot 10^6$ |

**Force** *F* [N]

| B \ A | oz | lbf | N | kp | p |
|---|---|---|---|---|---|
| N | 0.278 | 4.448 | – | 9.807 | $9.807 \cdot 10^{-3}$ |
| kp | 0.028 | 0.454 | 0.102 | – | $1 \cdot 10^{-3}$ |
| oz | – | 16 | 3.600 | 35.27 | $35.27 \cdot 10^{-3}$ |
| lbf | 1/16 | – | 0.225 | 2.205 | $2.205 \cdot 10^{-3}$ |
| pdl | 2.011 | 32.17 | 7.233 | 70.93 | $70.93 \cdot 10^{-3}$ |

**Length** *l* [m]

| B \ A | in | ft | yd | Mil | m | cm | mm | m |
|---|---|---|---|---|---|---|---|---|
| m | $25.4 \cdot 10^{-3}$ | 0.305 | 0.914 | $25.4 \cdot 10^{-6}$ | – | 0.01 | $1 \cdot 10^{-3}$ | $1 \cdot 10^{-6}$ |
| cm | 2.54 | 30.5 | 91.4 | $25.4 \cdot 10^{-4}$ | $1 \cdot 10^2$ | – | 0.1 | $1 \cdot 10^{-4}$ |
| mm | 25.4 | 305 | 914 | $25.4 \cdot 10^{-3}$ | $1 \cdot 10^3$ | 10 | – | $1 \cdot 10^{-3}$ |
| in | – | 12 | 36 | $1 \cdot 10^{-3}$ | 39.37 | 0.394 | $3.94 \cdot 10^{-2}$ | $3.94 \cdot 10^{-5}$ |
| ft | $^1/_{12}$ | – | 3 | $^1/_{12} \cdot 10^{-3}$ | 3.281 | $3.281 \cdot 10^{-2}$ | $3.281 \cdot 10^{-3}$ | $3.281 \cdot 10^{-6}$ |

**Angular velocity** *ω* [$s^{-1}$]

| B \ A | $s^{-1}$ = Hz | rpm | rad $s^{-1}$ |
|---|---|---|---|
| rad $s^{-1}$ | 2p | $^p/_{30}$ | – |
| rpm | $^1/_{60}$ | – | $^{30}/_p$ |

**Angular acceleration** *α* [$s^{-2}$]

| B \ A | $min^{-2}$ | $s^{-2}$ | rad $s^{-2}$ | rpm $s^{-2}$ |
|---|---|---|---|---|
| $s^{-2}$ | $^1/_{3600}$ | – | $^1/_{2p}$ | $^1/_{60}$ |
| rad $s^{-2}$ | $^p/_{1800}$ | 2p | – | $^p/_{30}$ |

**Linear velocity** *v* [m $s^{-1}$]

| B \ A | in-$s^{-1}$ | in-rpm | ft-$s^{-1}$ | ft-rpm | m $s^{-1}$ | cm $s^{-1}$ | mm $s^{-1}$ | m rpm |
|---|---|---|---|---|---|---|---|---|
| m $s^{-1}$ | $2.54 \cdot 10^{-2}$ | $4.23 \cdot 10^{-4}$ | 0.305 | $5.08 \cdot 10^{-3}$ | – | $1 \cdot 10^{-2}$ | $1 \cdot 10^{-3}$ | $^1/_{60}$ |
| in-$s^{-1}$ | – | 60 | 12 | 720 | 39.37 | $39.37 \cdot 10^{-2}$ | $39.37 \cdot 10^{-3}$ | 0.656 |
| ft-$s^{-1}$ | $^1/_{12}$ | 5 | – | 60 | 3.281 | $3.281 \cdot 10^{-2}$ | $3.281 \cdot 10^{-3}$ | $5.46 \cdot 10^{-2}$ |

**Temperature** *T* [K]

| B \ A | ° Fahrenheit | ° Celsius | Kelvin |
|---|---|---|---|
| Kelvin | (°F -305.15)/1.8 | °C + 273.15 | – |
| ° Celsius | (°F -32)/1.8 | – | K -273.15 |
| ° Fahrenheit | – | 1.8°C + 32 | 1.8 K + 305.15 |

Units used in the maxon catalog

## 10.2 Typical coefficients of friction for rolling, static and kinetic friction

| Type of friction | Friction condition | Description |
|---|---|---|
| Kinetic friction | Solid-to-solid friction (dry kinetic friction) | Direct contact between the friction partners |
| | Boundary friction (lubricated kinetic friction) | Special case of solid-to-solid friction with adsorbed lubricant on the surfaces |
| | Mixed friction | Solid-to-solid friction and fluid friction combined next to each other |
| | Fluid friction | Friction partners are completely separated from each other by a film of fluid (produced hydrostatically or hydrodynamically) |
| | Gas friction | Friction partners are completely separated from each other by a gas film (produced aerostatically or aerodynamically) |
| Static friction | | 20 ... 100% higher than kinetic friction |
| Rolling friction | Rolling friction | Bodies separated by lubricated roller bearings |
| | Combined rolling and sliding friction | Rolling friction with a kinetic component (slip) |

| Typical coefficient of friction | Examples | Coefficient of friction |
|---|---|---|
| 0.1 ... 1 | Sintered bronze – Steel | 0.15 ... 0.3 |
| | Plastic – Gray cast iron | 0.3 ... 0.4 |
| | Steel – Steel | 0.4 ... 0.7 |
| | Nitrided steel – Nitrided steel | 0.3 ... 0.4 |
| | Copper – Copper | 0.6 ... 1.0 |
| | Chromium – Chromium | 0.41 |
| | Al alloy – Al alloy | 0.15 ... 0.6 |
| 0.1 ... 0.2 | Steel – Steel | 0.1 |
| 0.01 ... 0.1 | Sleeve bearing, lubricated, at low speeds of rotation | |
| | Sintered bronze – Steel | 0.05 ... 0.1 |
| | Sintered iron – Steel | 0.07 ... 0.1 |
| | Tempered steel – Tempered steel | 0.05 ... 0.08 |
| 0.001 ... 0.01 | Sintered sleeve bearing, lubricated, at high speeds of rotation and low radial load | |
| 0.0001 | | |
| 0.1 ... 1.2 | Steel – Steel dry | 0.4 ... 0.8 |
| | Steel – Steel lubricated | 0.08 ... 0.12 |
| | Sintered bronze – Steel dry | 0.2 ... 0.4 |
| | Sintered bronze – Steel lubricated | 0.12 ... 0.14 |
| | Plastic – Gray cast iron, dry | 0.3 ... 0.5 |
| 0.001 ... 0.005 | Ball bearings | 0.001 ... 0.0025 |
| 0.001 ... 0.1 | | |

# 11. Symbol list for the Formulae Handbook

| Name | Symbol | Unit | Page number |
|---|---|---|---|
| Acceleration | $a$ | m/s² | 9, 14 |
| Acceleration force | $F_a$ | N | 9, 24 |
| Acceleration force, 1st / 2nd half cycle | $F_{a1}$ / $F_{a2}$ | N | 24 |
| Acceleration time | $\Delta t_a$ | s | 22, 23, 24, 25 |
| Acceleration time | $\Delta t$ | s | 41 |
| Ambient temperature | $T_A$ | °C | 48, 49, 50, 51, 52 |
| Angle of the inclined plane | $\alpha$ | ° | 9 |
| Angular acceleration | $\alpha$ | rad/s² | 11, 15, 41 |
| Angular frequency | $\omega$ | rad/s | 33 |
| Angular velocity / Angular velocity (change) | $\omega$ / $\omega$, $\Delta\omega$ / $\Delta\omega$ | rad/s | 11, 15, 21 |
| Angular velocity after / before acceleration | $\omega_{end}$ / $\omega_{start}$ | rad/s | 15 |
| Angular velocity, input / load | $\omega_{in}$ / $\omega_L$ | rad/s | 21 |
| Average end temperature, winding | $T_{W,av\infty}$ | °C | 51 |
| Bearing load, axial / radial | $F_{KL}$ | N | 11 |
| Capacitance | $C$ | F | 30, 33, 34 |
| Coefficient of friction (see table chapt. 10.2) | $\mu$ | | 9, 11 |
| Compressive force | $F_p$ | N | 9 |
| Counts per turn, CPT | $N$ | | 44 |
| Cross section | $A$ | m² | 9, 13 |
| Current | $I$ | A | 29, 33, 34, 48, 51 |
| Current change | $di$ | A | 40 |
| Current during ON phase | $I_{on}$ | A | 51 |
| Current ripple, peak-to-peak | $\Delta I_{PP}$ | A | 46 |
| Current through resistor $R_2$ | $I_{R2}$ | A | 32 |
| Cut-off frequency | $f_C$ | Hz | 34 |
| Density | $\rho$ | kg/m³ | 12, 13 |
| Diameter, deflector pulley 2 | $d_2$ | m | 22 |
| Diameter, deflector pulley $X$ | $d_X$ | m | 22 |
| Diameter, drive pulley | $d_1$ | m | 22, 25 |
| Diameter, drive wheel | $d$ | m | 23 |
| Diameter, load pulley | $d_2$ | m | 25 |
| Displacement | $\Delta_l$ | m | 9 |
| Distance change | $\Delta s$ | m | 14, 16, 17 |
| Distance of axis $s$ from center of gravity $S$ | $r_s$ | m | 13 |
| Downhill-slope force | $F_H$ | N | 9 |
| Drop height | $h$ | m | 14 |
| Duration | $\Delta t$ | s | 9, 11 |
| Duration of operating points 1…$n$ | $t_{1\ldots n}$ | s | 42 |
| Eccentricity | $e$ | m | 24 |
| Eddy current power losses | $P_{V,eddy}$ | W | 47 |
| Efficiency | $\eta$ | | 21, 22, 23, 24, 25, 40, 47 |
| Electric charge | $Q$ | C | 48 |
| Electrical capacitance | $C_1$ / $C_2$ | F | 48 |
| Electrical input power | $P_{el}$ | W | 38 |
| Electrical resistance | $R$ / $R_1$ / $R_2$ | Ω | 29, 30, 33, 34, 48 |
| Electrical time constant | $\tau_{el}$ | s | 30 |
| Electromotive force | $EMK$ | V | 40 |
| End temperature | $T_{end}$ | °C | 48 |
| End temperature, stator / winding | $T_{S,\infty}$ / $T_{W,\infty}$ | °C | 48, 49, 50, 51, 52 |
| Equivalent resistance | $R$ | Ω | 31, 32 |
| Equivalent resistance of $R_2$ and $R_L$ | $R_x$ | Ω | 32 |
| Force | $F$ | N | 9, 11, 21 |
| Frequency | $f$ | Hz | 33, 34 |
| Friction force | $F_R$ | N | 9 |
| Friction power losses | $P_{V,R}$ | W | 47 |
| Friction torque | $M_R$ | Nm | 11, 39, 40, 41 |
| Gearhead efficiency | $\eta_G$ | | 47 |
| Gearhead temperature | $T_G$ | °C | 49 |
| Gravitational acceleration | $g$ | m/s² | 9, 14 |
| Ground | $GND$ | V | 48 |
| Heat capacity | $C_{th}$ | J/K | 47 |
| Heat capacity gearhead / stator / winding | $C_{th,G}$ / $C_{th,S}$ / $C_{th,W}$ | J/K | 47, 48 |
| Height | $h$ | m | 12, 13 |
| Impedance | $Z$ | Ω | 33 |
| Induced voltage | $U_{ind}$ | V | 39, 40 |
| Inductance | $L$ | H | 30, 33, 34 |
| Inductance, additional external motor choke | $L_{ext}$ | H | 46 |
| Inductance, built-in choke controller | $L_{int}$ | H | 46 |
| Inner radius | $r_i$ | m | 12, 13 |
| Inner resistance, voltage source | $R_i$ | Ω | 29 |
| Input speed | $n_{in}$ | rpm | 22, 23, 24, 25 |
| Input torque | $M_{in}$ | Nm | 21, 22, 23, 25 |
| Input voltage | $U_{in}$ | V | 30, 34 |
| Joule power loss | $P_J$ | W | 38, 47, 49, 52 |
| Length | $l$ | m | 13 |
| Length of side $a$ / $b$ / $c$ | $a$ / $b$ / $c$ | m | 13 |
| Load current | $I_L$ | A | 32 |
| Load force (output) | $F_L$ | N | 10, 21, 22, 23 |
| Load force, 1st / 2nd half cycle | $F_{L1}$ / $F_{L2}$ | N | 24 |
| Load resistance | $R_L$ | Ω | 29, 32 |
| Load speed | $n_L$ | rpm | 25, 41, 45, 47 |
| Load torque | $M_L$ | Nm | 11, 21, 25, 41, 45, 47 |
| Load velocity | $v_L$ | m/s | 21, 22, 23 |
| Load voltage | $U_L$ | V | 32 |

| Name | Symbol | Unit | Page number |
|---|---|---|---|
| Mass | $m$ | kg | 9, 12, 13, 47 |
| Mass, belt | $m_B$ | kg | 22 |
| Mass, belt | $m_R$ | kg | 25 |
| Mass, gear rack | $m_Z$ | kg | 23 |
| Mass, gearhead | $m_G$ | kg | 47 |
| Mass, motor | $m_{mot}$ | kg | 47 |
| Mass of the load | $m_L$ | kg | 22, 23, 24 |
| Mass, rover | $m_F$ | kg | 23 |
| Mass, screw nut | $m_S$ | kg | 22 |
| Mass, winding | $m_W$ | kg | 47 |
| Maximum acceleration | $\alpha_{max}$ | m/s² | 16, 17 |
| Maximum angular acceleration | $\alpha_{max}$ | rad/s² | 18, 19, 41 |
| Maximum continuous torque, gearhead (catalog value) | $M_{G,cont}$ | Nm | 26, 47 |
| Maximum current ripple, peak-to-peak | $\Delta I_{PP,max}$ | A | 46 |
| Maximum efficiency at $U_N$ (catalog value) | $\eta_{max}$ | | 40 |
| Maximum input speed (catalog value) | $n_{in,max}$ | rpm | 26 |
| Maximum intermittent torque, gearhead (catalog value) | $M_{G,max}$ | Nm | 26 |
| Maximum output speed gearhead | $n_{G,max}$ | rpm | 26 |
| Maximum power | $P_{max}$ | W | 29 |
| Maximum speed in load cycle | $n_{max}$ | rpm | 18, 19, 42 |
| Maximum temperature change | $\Delta T_{max}$ | K | 48 |
| Maximum torque in load cycle | $M_{max}$ | Nm | 42 |
| Maximum velocity | $v_{max}$ | m/s | 16, 17 |
| Maximum voltage drop of the controller | $\Delta U_{max}$ | V | 45 |
| Maximum winding temperature (catalog value) | $T_{max}$ | °C | 50, 51, 52 |
| Mean radius bearing | $r_{KL}$ | m | 11 |
| Measurement resolution | $\Delta Q$ | qc/ms | 44 |
| Measurement resolution, motor speed | $\Delta n$ | rpm | 44 |
| Mechanical input power | $P_{in,mech}$ | W | 21 |
| Mechanical output power | $P_L$ / $P_{L,mech}$ | W | 21, 38 |
| Mechanical play, input | $\Delta\varphi_{in}$ | rad | 22, 23, 25 |
| Mechanical play, load | $\Delta s_L$ / $\Delta\varphi_L$ | m/rad | 22, 23, 25 |
| Mechanical power | $P_{mech}$ | W | 21 |
| Mechanical time constant (catalog value) | $\tau_m$ | s | 41 |
| Mechanical time constant with additional $J_L$ | $\tau_m'$ | s | 41 |
| Moment of inertia | $J$ | kgm² | 11 |
| Moment of inertia with reference to the axis *s* through the center of gravity *S* | $J_s$ | kgm² | 13 |
| Moment of inertia with reference to the rotation axis *x* | $J_x$ | kgm² | 12, 13 |
| Moment of inertia with reference to the rotation axis *y* | $J_y$ | kgm² | 12, 13 |
| Moment of inertia with reference to the rotation axis *z* | $J_z$ | kgm² | 12, 13 |
| Moment of inertia, all wheels together | $J_W$ | kgm² | 23 |
| Moment of inertia, deflector pulley 2/*X* | $J_2$ / $J_X$ | kgm² | 22, 25 |
| Moment of inertia, driving end | $J_1$ | kgm² | 22, 25 |
| Moment of inertia, eccentric disc | $J_E$ | kgm² | 24 |
| Moment of inertia, gearhead transformed | $J_G$ | kgm² | 25 |
| Moment of inertia, input (motor, encoder, brake) | $J_{in}$ | kgm² | 22, 23, 24, 25 |
| Moment of inertia, load | $J_L$ | kgm² | 25, 41 |
| Moment of inertia, load | $J_2$ | kgm² | 25 |
| Moment of inertia, pinion | $J_P$ | kgm² | 23 |
| Moment of inertia, rotor (catalog value) | $J_R$ | kgm² | 41 |
| Moment of inertia, screw | $J_S$ | kgm² | 22 |
| Motor current | $I_{mot}$ / $i_{mot}$ | A | 38, 39, 40, 41, 42, 47, 49, 50, 52 |
| Motor speed | $n_{mot}$ | rpm | 47 |
| Motor torque | $M_{mot}$ | Nm | 47 |
| Motor voltage | $U_{mot}$ | V | 38, 39, 40, 42, 47 |
| No load current | $I_0$ | A | 39, 40, 42 |
| No load speed | $n_0$ | rpm | 38, 39, 40, 41, 50 |
| No load speed of motor at $U_N$ (catalog value) | $n_{0,UN}$ | rpm | 45 |
| Nominal current as a function of $T_A$ | $I_{N,TA}$ | A | 50, 51 |
| Nominal current, motor (catalog value) | $I_N$ | A | 46, 50, 51, 52 |
| Nominal speed, motor (catalog value) | $n_N$ | rpm | 50 |
| Nominal torque, motor (catalog value) | $M_N$ | Nm | 42, 50 |
| Nominal voltage, motor (catalog value) | $U_N$ | V | 38, 39, 40, 45 |
| Normal force (force perpendicular to the surface of contact) | $F_N$ | N | 9 |
| Number of teeth, internal gear/pinion/sun wheel | $z_3$ / $z$ / $z_1$ | | 23, 25 |
| OFF time | $t_{off}$ | s | 51 |
| ON time | $t_{on}$ | s | 51, 52 |
| Outer radius | $r_a$ | m | 12, 13 |
| Output current | $I_{out}$ | A | 30 |
| Output voltage | $U_{out}$ | V | 30, 34 |
| Overload factor | $K$ | | 52 |
| Partial currents | $I_1$, $I_2$ | A | 31 |
| Partial forces | $F_1$ / $F_2$ / $F_x$ | N | 10 |
| Partial resistances | $R_1$, $R_2$ | Ω | 31, 32 |
| Partial torques | $M_1$ / $M_2$ / $M_x$ | Nm | 11 |
| Partial voltages | $U_1$, $U_2$ | V | 31, 32 |
| Period | $T$ | s | 33 |
| Periodic acceleration force as a function of the angle of rotation | $F_a(\varphi)$ | N | 24 |
| Phase shift | $\varphi$ | ° | 34 |
| Pitch | $p$ | m | 23 |
| Point 1/2 on the *x*-axis | $x_1$ / $x_2$ | m | 13 |
| Position resolution | $\Delta\varphi$ | ° | 44 |
| Potentiometer position | $x$ | 0…1 | 32 |
| Power | $P$ | W | 29, 38 |
| Power losses/power losses, gearhead | $P_V$ / $P_{V,G}$ | W | 29, 48, 49, 50 |
| Power losses for reversal of magnetization | $P_{V,magn}$ | W | 47 |

| Name | Symbol | Unit | Page number |
|---|---|---|---|
| Pressure (1 Pa = 1 N/m² = 10⁻⁵ bar) | $p$ | Pa | 9 |
| Pull-down resistance | $R_d$ | Ω | 30 |
| Pull-up resistance | $R_u$ | Ω | 30 |
| PWM frequency | $f_{PWM}$ | Hz | 46 |
| Quadcounts per pulse | $Q = 4$ | qc/IMP | 44 |
| Radius / Radius 1 / Radius 2 | $r$ / $r_1$ / $r_2$ | m | 11, 12 |
| Radius circular torus around *z*-axis | $R$ | m | 12 |
| Reactance, capacitive | $X_C$ | Ω | 33, 34 |
| Reactance, inductive | $X_L$ | Ω | 33, 34 |
| Reduction ratio, gearhead (catalog value) | $i_G$ | | 25, 26 |
| Reduction ratio, mechanical drive | $i$ | | 44 |
| Required no load speed | $n_{0,theor}$ | rpm | 42 |
| Required speed constant | $k_{n,theor}$ | rpm/V | 42 |
| Resistance coefficient, copper | $\alpha_{Cu}$ | 0.0039 $K^{-1}$ | 32, 47, 49, 50, 51, 52 |
| Resistance, potentiometer | $R_0$ | Ω | 32 |
| RMS current | $I_{RMS}$ | A | 51 |
| RMS torque | $M_{in,RMS}$ / $M_{RMS}$ | Nm | 24, 42 |
| Rotation angle / rotation angle change | $\varphi$ / $\Delta\varphi$ | rad | 11, 15, 18, 19, 24 |
| Screw lead (pitch) | $p$ | m | 22 |
| Sinusoidal velocity curve of the load | $v_L(t)$ | m/s | 24 |
| Source voltage | $U_0$ | V | 29 |
| Specific heat capacity | $c$ | J/(kgK) | 47 |
| Specific heat capacity copper | $c_{Cu}$ | 380 J/(kgK) | 47 |
| Specific heat capacity iron | $c_{Fe}$ | 450 – 470 J/(kgK) | 47 |
| Speed / torque gradient, motor (catalog value) | $\Delta n$ / $\Delta M$ | rpm/mNm | 39, 40, 42, 45 |
| Speed after acceleration | $n_{end}$ | rpm | 15 |
| Speed before acceleration | $n_{start}$ | rpm | 15 |
| Speed change | $\Delta n$ | rpm | 11 |
| Speed change, input | $\Delta n_{in}$ | rpm | 22, 23, 24, 25 |
| Speed constant (catalog value) | $k_n$ | rpm/V | 39, 40, 42 |
| Speed of rotation / Speed of rotation (change) | $n$ / $n$, $\Delta n$ | rpm | 15, 21, 38 ,39, 40, 41, 42, 45, 47, 50 |
| Spring constant | $k$ | N/m | 9 |
| Spring force | $F_S$ | N | 9 |
| Stall torque | $M_H$ | Nm | 38, 39, 40, 41, 42 |
| Stall torque at temperature $T_W$ | $M_{HT}$ | Nm | 47 |
| Stands for $X_C$ or $X_L$ | $X$ | Ω | 33 |
| Starting current | $I_A$ | A | 39, 40 |
| Starting current at temperature $T_W$ | $I_{AT}$ | A | 47 |
| Stored heat | $Q$ | J | 47, 48 |
| Supply voltage | $+V$ | V | 30 |
| Supply voltage | $V_{CC}$ | V | 45, 46 |
| Temperature | $T$ | °C | 47, 48, 49, 51, 52 |
| Temperature at start | $T_{start}$ | °C | 48 |
| Temperature change as a function of time *t* | $\Delta T(t)$ | K | 48 |
| Temperature difference | $\Delta T$ | K | 47 |
| Temperature difference, gearhead–ambient | $\Delta T_G$ | K | 49 |
| Temperature difference, stator–ambient | $\Delta T_{S,\infty}$ | K | 48, 49, 50, 51 |
| Temperature difference, winding–ambient | $\Delta T_{W,\infty}$ | K | 48, 49, 51, 52 |
| Terminal inductance, motor (catalog value) | $L_{mot}$ | H | 40, 46 |
| Terminal resistance, motor (catalog value) | $R_{mot}$ | Ω | 32, 38, 39, 40, 41, 47 |
| Terminal voltage | $U_{kl}$ | V | 29 |
| Therm. resistance, gearhead–ambient | $R_{th,G}$ | K/W | 49 |
| Therm. resistance, housing–ambient (catalog value) | $R_{th2}$ | K/W | 48, 49, 50, 51, 52 |
| Therm. resistance, housing–ambient modified | $R_{th2,mod}$ | K/W | 50 |
| Therm. resistance, winding–housing (catalog value)– | $R_{th1}$ | K/W | 48, 49, 50, 51, 52 |
| Therm. time constant, motor / winding (catalog value) | $\tau_M$ / $\tau_W$ | s | 49, 52 |
| Thermal time constant | $\tau_{th}$ | s | 48 |
| Time / Time, duration | $t$ / $t$, $\Delta t$ | s | 14, 15, 24, 30, 33, 41, 47, 48, 49, 51, 52 |
| Time *a* / *b* / *c* | $\Delta t_a$ / $\Delta t_b$ / $\Delta t_c$ | s | 16, 18 |
| Time change | $dt$ | s | 40 |
| Torque | $M$ | Nm | 11, 21, 38, 39, 40, 41, 42, 45, 47, 50 |
| Torque at operating points 1…*n* | $M_{1 \ldots n}$ | Nm | 42 |
| Torque constant (catalog value) | $k_M$ | Nm/A | 39, 40, 41, 42, 47 |
| Torque for acceleration | $M_{in,\alpha}$ / $M\alpha$ | Nm | 11, 22, 23, 24, 25 |
| Torque for reversal of magnetization | $M_{magn}$ | Nm | 47 |
| Torque, 1st / 2nd half cycle | $M_{in1}(\varphi)$ / $M_{in2}(\varphi)$ | Nm | 24 |
| Torque, spiral spring | $M_S$ | Nm | 11 |
| Torsion coefficient (spring constant) | $k_m$ | Nm | 11 |
| Total current | $I$ | A | 31, 32 |
| Total inductance | $L_{tot}$ | H | 46 |
| Total time | $\Delta t_{tot}$ | s | 16, 17, 18, 19 |
| Total time, operating cycle | $t_{tot}$ | s | 42 |
| Total voltage | $U$ | V | 31, 32 |
| Velocity / Velocity (change) | $v$ / $v$, $\Delta v$ / $\Delta v$ | m/s | 9, 14, 21 |
| Velocity after acceleration | $v_{end}$ | m/s | 14 |
| Velocity before acceleration | $v_{start}$ | m/s | 14 |
| Voltage | $U$ | V | 29, 33 |
| Voltage over capacitance / inductance/resistance | $U_C$ / $U_L$ / $U_R$ | V | 34 |
| Voltage, potential difference | $U_1$ / $U_2$ | V | 48 |
| Weight of a body | $F_G$ | N | 9 |
| Winding resistance at current temperature $T_W$ | $R_{TW}$ | Ω | 32, 47 |
| Winding resistance at temperature $T_A$ | $R_{TA}$ | Ω | 49, 50, 51, 52 |
| Winding temperature | $T_W$ | °C | 32, 47 |